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# A Minimal Superstring Standard Model I: Flat Directions

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## Abstract

Three family  $SU(3)_C \times SU(2)_L \times U(1)_Y$  string models in several constructions generically possess two features: (i) an extra local anomalous  $U(1)_A$  and (ii) numerous (often fractionally charged) exotic particles beyond those in the minimal supersymmetric model (MSSM). Recently, we demonstrated that the observable sector effective field theory of such a free fermionic string model can reduce to that of the MSSM, with the standard observable gauge group being just  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and the  $SU(3)_C \times SU(2)_L \times U(1)_Y$ -charged spectrum of the observable sector consisting solely of the MSSM spectrum. An example of a model with this property was shown. We continue our investigation of this model by presenting a large set of different flat directions of the same model that all produce the MSSM spectrum. Our results suggest that even after imposing the conditions for the decoupling of exotic states, there may remain sufficient freedom to satisfy the remaining phenomenological constraints imposed by the observed data.

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# 1 Introduction

Deriving the Standard Model from heterotic string theory remains one of the strongly motivated endeavours in theoretical physics. On the one hand, for over a quarter of a century now, the structure of the Standard model itself suggests the realization of grand unified structures, most appealing in the context of  $SO(10)$  unification [1, 2]. On the other hand supersymmetry, a key ingredient in grand and string unification, continues to be the only extension of the Standard Model still consistent with the experimental data, whereas theories with a low energy cutoff in general run into conflict with experiment once detailed models are considered. Finally, string theory remains the only known theoretical framework for the consistent unification of gravity and the gauge interactions.

There are two main approaches regarding how to derive the Standard Model from string theory. The first approach asserts that we must first understand the non-perturbative formulation of string theory and then the true string vacuum will be uniquely revealed. The second approach argues that much can be learned about the string realization of the Standard Model by studying the features and properties of phenomenologically promising perturbative string models. Singling out such string models for study may be instrumental for learning about the nonperturbative dynamics of the theory. Following the second line of thought, a class of phenomenologically promising models has been identified. These models, constructed in the free fermionic formulation [3], correspond to  $Z_2 \times Z_2$  orbifold models with nontrivial Wilson lines and background fields. Studying F-theory compactification on the  $Z_2 \times Z_2$  orbifold, which is related to the free fermionic models [4], indeed reveals new features that do not arise in other similar F-theory studies [5]. It is not unlikely that these new features will eventually prove to be important for understanding the issue of vacuum selection in string theory.

Connecting string theory to low energy experimental data, parameterized by the Standard Model, remains vital. It is important to point out that of the semi-realistic orbifold [6] and free fermionic models [7, 8, 9, 10, 11, 12], the NAHE-based  $Z_2 \times Z_2$  free fermionic models [13, 10] (or more generally  $Z_2 \times Z_2$  orbifold models at the free fermionic point) are the only ones that naturally give rise to the  $SO(10)$  unification structures. Thus, it seems of much value to continue improving our understanding of the more phenomenologically viable three generation  $Z_2 \times Z_2$  free fermionic models, as well as their embeddings in compactifications of M- and F-theory.

It was stressed in the past for a variety of phenomenological reasons that the canonical  $SO(10)$  embedding of the weak-hypercharge in string models is highly preferred [14]. In fact, it was even suggested that this must be the case in the true string vacuum. We stress that this is precisely the embedding obtained in the  $Z_2 \times Z_2$  free fermionic models, in contrast to other quasi-realistic orbifold [6] or free fermionic models [12], which do not produce the standard  $SO(10)$  embedding. The first phenomenological criterion that a string model must satisfy are three chiral

generations with the standard  $SO(10)$  embedding of the Standard Model gauge group. While the  $Z_2 \times Z_2$  free fermionic models naturally give rise to the  $SO(10)$  unification structure, the  $SO(10)$  symmetry has to be broken at the string level. Therefore, there is no explicit  $SO(10)$  symmetry in the effective field theory level. Nevertheless, the  $SO(10)$  symmetry is still reflected, for example, in some of the Yukawa coupling relations.

In realistic heterotic string models, it is well known that modular invariance constraints impose that the string spectrum contains exotic fractionally charged states [15]. Such states indeed occur in all known string models built from level-one Kač–Moody algebras, and may appear in the massive or massless sectors. In many of the more realistic examples such states arise in vector-like representations and therefore may obtain mass terms from cubic-level or higher order terms in the superpotential. It is clear that in the true string vacuum such fractionally charged states must be either confined [7] or sufficiently massive [16, 17, 18]. Thus, the next non-trivial phenomenological criteria on viable string models is that there should be no free fractionally charged states surviving to low energies. Furthermore, as the existence of fractionally charged states and other states beyond the minimal supersymmetric standard model will in general affect the unification of the gauge couplings, an attractive scenario is that all the states beyond the MSSM decouple from the low energy spectrum at the string scale.

Recently, we have demonstrated the existence of string models with the above properties [19]. Studying the flat directions in the string model of ref. [8] (referred to henceforth as the “FNY model”), we showed the existence of one flat direction for which the massless spectrum below the string scale consists solely of the spectrum of the minimal supersymmetric standard model. Furthermore, it was found that in this particular model the  $D$ -flatness constraints necessarily impose that either of the  $U(1)_{Z'}$  or  $U(1)_Y$  symmetries must be broken by the choices of flat directions. This is in fact an attractive situation in which in the phenomenologically viable case the surviving  $SO(10)$  subgroup necessarily coincides with that of the Standard Model, and the  $U(1)_{Z'}$  is necessarily broken by the choices of flat directions.

We remark that our model also contains, at the massless string level, a number of electroweak Higgs doublets and a color triplet/anti-triplet pair beyond the MSSM. We show that by the same suitable choices of flat directions that only one Higgs pair remains light below the string scale. The additional color triplet/anti-triplet pair receives mass from a fifth order superpotential term. This results in the triplet pair receiving a mass that is slightly below the string scale and is perhaps smaller than the doublet and fractional exotic masses by a factor of around  $(1/10 - 1/100)$ . We emphasize that the numerical estimate of the masses arising from the singlet VEVs should be regarded only as illustrative. The important result is the generation of mass terms for all the states beyond the MSSM, near the string scale. The actual masses of the extra fields may be spread around the  $M_U$  scale, thus inducing small threshold corrections that are still expected to be compatible with the low energy

experimental data.

The string solution found in ref. [19] is the first known example of a minimal superstring derived standard model, in which, of all the  $SU(3)_C \times SU(2)_L \times U(1)_Y$ -charged states, only the MSSM spectrum remains light below the string scale. In this paper we expand the analysis of ref. [19]. The first obvious question is naturally whether the special solution found in ref. [19] is an isolated example or whether there exists an enlarged space of solutions giving rise solely to the MSSM spectrum below the string scale. We recall that in the FNY model of ref. [8] the condition for the decoupling of the exotic fractionally charged states from the massless spectrum is that a specific set of Standard Model singlet fields [16] acquire non-vanishing vacuum expectation values (VEVs) in the cancellation of the anomalous  $U(1)_A$  Fayet–Iliopoulos (FI)  $D$ -term. Additionally we showed that the same set of VEVs produce mass terms that lead to the decoupling of the extra color triplets and electroweak doublets, beyond the MSSM. Thus, in any solution that incorporates those VEVs, the resulting spectrum below the string scale consists solely of the MSSM spectrum. Expanding the analysis of ref. [19] we show that there exists, in fact, an extended space of solutions which incorporate those VEVs. This is a very promising situation, for it demonstrates that there may still be sufficient freedom to allow the possibility of accommodating the various phenomenological constraints in one of these solutions.

Our paper is organized as follows: Section 2 is a brief review of the FNY model [8], including a discussion of the model’s massless spectrum, prior to any states taking on VEVs. This is accompanied by Appendix A, which lists all string quantum numbers, including the non-gauge charges, of the massless states, and by Appendix B, which contains the complete renormalizable superpotential and all fourth through sixth order non-renormalizable superpotential terms. Section 3 reviews constraints on flat directions and presents a survey of viable flat directions that generate (near) string-scale mass to all  $SU(3)_C \times SU(2)_L \times U(1)_Y$ -charged MSSM exotic states in the FNY model, producing an effective MSSM field theory below the string scale. Accompanying Section 3 are Appendices C and D. Appendix C contains a SM-conserving basis set, of the “maximally orthogonal” class, for generating  $D$ -flat directions, while Appendix D contains the actual tables of classes of  $D$ - and  $F$ -flat directions. These tables indicate the order to which  $F$ -flatness is retained, the respective superpotential terms that break  $F$ -flatness, the dimension of each flat direction, and the respective number of non-anomalous  $U(1)_i$  that are broken by these directions. Section 4 then discusses some distinguishing features of the various flat directions. Phenomenological implications of these flat directions will be presented in [20].

## 2 The FNY Model (A Review)

## 2.1 Construction

The more realistic free fermionic models, which utilize the NAHE-set of boundary condition basis vectors, admit the  $SO(10)$  embedding of the Standard Model gauge group. Aside from the phenomenological aspects, which motivate the need for the  $SO(10)$  embedding, another advantage of utilizing the standard  $SO(10)$  embedding and hence of the NAHE-set, is the decoupling [21] of the exotic fractionally charged states from the massless spectrum. This should be contrasted with models, like the free fermionic models of [12], which do not allow the  $SO(10)$  embedding and contain exotic fractionally charged states which cannot decouple from the massless spectrum [22]. This distinction is an important one, as it severely limits the number of phenomenologically viable models, even among the three generation orbifold models that are traditionally viewed as semi-realistic. The more realistic NAHE-based free fermionic models represent one class of string models that still survives this requirement.

For completeness we recall here the construction of the FNY model and its distinctive properties. The boundary condition basis vectors which generate the more realistic free fermionic models are, in general, divided into two major subsets. The first set consists of the NAHE set [13, 10], which is a set of five boundary condition basis vectors denoted  $\{\mathbf{1}, \mathbf{S}, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ . With ‘0’ indicating Neveu–Schwarz boundary conditions and ‘1’ indicating Ramond boundary conditions, these vectors are as follows:

	$\psi^\mu$	$\chi^{12}$	$\chi^{34}$	$\chi^{56}$	$\bar{\psi}^{1,\dots,5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1,\dots,8}$
<b>1</b>	1	1	1	1	1,...,1	1	1	1	1,...,1
<b>S</b>	1	1	1	1	0,...,0	0	0	0	0,...,0
<b>b<sub>1</sub></b>	1	1	0	0	1,...,1	1	0	0	0,...,0
<b>b<sub>2</sub></b>	1	0	1	0	1,...,1	0	1	0	0,...,0
<b>b<sub>3</sub></b>	1	0	0	1	1,...,1	0	0	1	0,...,0

  

	$y^{3,\dots,6}$	$\bar{y}^{3,\dots,6}$	$y^{1,2}, \omega^{5,6}$	$\bar{y}^{1,2}, \bar{\omega}^{5,6}$	$\omega^{1,\dots,4}$	$\bar{\omega}^{1,\dots,4}$
<b>1</b>	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1	1,...,1
<b>S</b>	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0	0,...,0
<b>b<sub>1</sub></b>	1,...,1	1,...,1	0,...,0	0,...,0	0,...,0	0,...,0
<b>b<sub>2</sub></b>	0,...,0	0,...,0	1,...,1	1,...,1	0,...,0	0,...,0
<b>b<sub>3</sub></b>	0,...,0	0,...,0	0,...,0	0,...,0	1,...,1	1,...,1

(2.1)

with the following choice of phases which define how the generalized GSO projections are to be performed in each sector of the theory:

$$C \begin{pmatrix} \mathbf{b}_i \\ \mathbf{b}_j \end{pmatrix} = C \begin{pmatrix} \mathbf{b}_i \\ \mathbf{S} \end{pmatrix} = C \begin{pmatrix} \mathbf{1} \\ \mathbf{1} \end{pmatrix} = -1. \quad (2.2)$$

The remaining projection phases can be determined from those above through the self-consistency constraints. The precise rules governing the choices of such vectors and phases, as well as the procedures for generating the corresponding space-time particle spectrum, are given in refs. [3].

After imposing the NAHE set, the resulting model has gauge group  $SO(10) \times SO(6)^3 \times E_8$  and  $N = 1$  space-time supersymmetry. The model contains 48 multiplets in the 16 representation of  $SO(10)$ , 16 from each twisted sector  $\mathbf{b}_1$ ,  $\mathbf{b}_2$  and  $\mathbf{b}_3$ . In addition to the spin 2 multiplets and the space-time vector bosons, the untwisted sector produces six multiplets in the vectorial 10 representation of  $SO(10)$  and a number of  $SO(10) \times E_8$  singlets. As can be seen from Table (2.1), the model at this stage possesses a cyclic permutation symmetry among the basis vectors  $\mathbf{b}_1$ ,  $\mathbf{b}_2$  and  $\mathbf{b}_3$ , which is also respected by the massless spectrum.

The second stage in the construction of these NAHE-based free fermionic models consists of adding three additional basis vectors to the above NAHE set. These three additional basis vectors, which are often called  $\{\alpha, \beta, \gamma\}$ , correspond to “Wilson lines” in the orbifold construction. The allowed fermion boundary conditions in these additional basis vectors are of course also constrained by the string consistency constraints, and must preserve modular invariance and world-sheet supersymmetry. The choice of these additional basis vectors  $\{\alpha, \beta, \gamma\}$  nevertheless distinguishes between different models and determine their low-energy properties. For example, three additional vectors are needed to reduce the number of massless generations to three, one from each sector  $\mathbf{b}_1$ ,  $\mathbf{b}_2$ , and  $\mathbf{b}_3$ , and the choice of their boundary conditions for the internal fermions  $\{y, \omega | \bar{y}, \bar{\omega}\}^{1, \dots, 6}$  also determines the Higgs doublet-triplet splitting and the Yukawa couplings. These low-energy phenomenological requirements therefore impose strong constraints [10] on the possible assignment of boundary conditions to the set of internal world-sheet fermions  $\{y, \omega | \bar{y}, \bar{\omega}\}^{1, \dots, 6}$ .

Consider the model in Table (2.3)

	$\psi^\mu$	$\chi^{12}$	$\chi^{34}$	$\chi^{56}$	$\bar{\psi}^{1, \dots, 5}$	$\bar{\eta}^1$	$\bar{\eta}^2$	$\bar{\eta}^3$	$\bar{\phi}^{1, \dots, 8}$
$\mathbf{b}_4$	1	1	0	0	1 1 1 1 1	1	0	0	0 0 0 0 0 0 0 0
$\beta$	1	0	0	1	1 1 1 0 0	1	0	1	1 1 1 1 0 0 0 0
$\gamma$	1	0	1	0	$\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} 0 1 1 \frac{1}{2} \frac{1}{2} \frac{1}{2} 1$

  

	$y^3 y^6$	$y^4 \bar{y}^4$	$y^5 \bar{y}^5$	$\bar{y}^3 \bar{y}^6$	$y^1 \omega^6$	$y^2 \bar{y}^2$	$\omega^5 \bar{\omega}^5$	$\bar{y}^1 \bar{\omega}^6$	$\omega^1 \omega^3$	$\omega^2 \bar{\omega}^2$	$\omega^4 \bar{\omega}^4$	$\bar{\omega}^1 \bar{\omega}^3$
$\mathbf{b}_4$	1	0	0	1	0	0	1	0	0	0	1	0
$\beta$	0	0	0	1	0	1	0	1	1	0	1	0
$\gamma$	0	0	1	1	1	0	0	1	0	1	0	0

(2.3)

With the choice of generalized GSO coefficients:

$$C \left( \begin{smallmatrix} \mathbf{b}_4 \\ \mathbf{b}_j, \beta \end{smallmatrix} \right) = -C \left( \begin{smallmatrix} \mathbf{b}_4 \\ \mathbf{1} \end{smallmatrix} \right) = -C \left( \begin{smallmatrix} \beta \\ \mathbf{1} \end{smallmatrix} \right) = C \left( \begin{smallmatrix} \beta \\ \mathbf{b}_j \end{smallmatrix} \right) =$$

$$-C\left(\begin{smallmatrix}\beta\\ \gamma\end{smallmatrix}\right) = C\left(\begin{smallmatrix}\gamma\\ \mathbf{b}_2\end{smallmatrix}\right) = -C\left(\begin{smallmatrix}\gamma\\ \mathbf{b}_1, \mathbf{b}_3, \mathbf{b}_4, \gamma\end{smallmatrix}\right) = -1$$

( $j = 1, 2, 3$ ), with the others specified by modular invariance and space–time supersymmetry. Several properties of the boundary conditions, eq. (2.3), that generate the FNY model distinguish it from other standard–like models [9]. First the basis vector  $\alpha \equiv \mathbf{b}_4$  does not break the  $SO(10)$  symmetry. In the models [9] both the basis vectors  $\alpha$  and  $\beta$  break the  $SO(10)$  symmetry to  $SO(6) \times SO(4)$ . This has the consequence that the combination  $\alpha + \beta$  gives rise to states transforming only under the observable  $(SO(10) \times SO(6)^3)$  part of the gauge group, which produce electroweak Higgs representations. Thus, we may anticipate that the structure of the Higgs mass matrix as well the fermion mass matrices in the FNY model will differ from those in the models of ref. [9]. Another distinction between the models is in the pairing of left–moving and right–moving fermions which produces Ising model operators [10].

## 2.2 Gauge Group

Before cancellation of the FI term by an  $F$ – and  $D$ –flat direction, the observable gauge group for the FNY model consists of the universal  $SO(10)$  sub–group,  $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$ , generated by the five complex world–sheet fermions  $\bar{\psi}^{1,\dots,5}$ , and six observable horizontal, flavor–dependent,  $U(1)$  symmetries  $U(1)_{1,\dots,6}$ , generated by  $\{\bar{\eta}^1, \bar{\eta}^2, \bar{\eta}^3, \bar{y}^3 \bar{y}^6, \bar{y}^1 \bar{\omega}^6, \bar{\omega}^1 \bar{\omega}^3\}$ , respectively. The hidden sector gauge group is the  $E_8$  sub–group of  $(SO(4) \sim SU(2) \times SU(2)) \times SU(3) \times U(1)^4$ , generated by  $\bar{\phi}^{1,\dots,8}$ .

The weak hypercharge is given by

$$U(1)_Y = \frac{1}{3}U(1)_C \pm \frac{1}{2}U(1)_L, \quad (2.4)$$

which has the standard effective level  $k_1$  of  $5/3$ , necessary for MSSM unification at  $M_U$ . As we noted in [19], the sign ambiguity in eq. (2.4) can be understood in terms of the two alternative embeddings of  $SU(5)$  within  $SO(10)$ , that produce either the standard or flipped  $SU(5)$  [23]. Switching signs in (2.4) flips the representations,

$$+ \leftrightarrow - \quad (2.5)$$

$$\begin{aligned} e_L^c &\leftrightarrow N_L^c \\ u_L^c &\leftrightarrow d_L^c \\ h &\leftrightarrow \bar{h} . \end{aligned} \quad (2.6)$$

In the case of  $SU(5)$  string GUT models, only the “–” (i.e., flipped version) is allowed, since there are no massless matter adjoint representations, which are needed to break the non–Abelian gauge symmetry of the unflipped  $SU(5)$ , but are not needed for the flipped version. For MSSM–like strings, either choice of sign is allowed since the GUT non–Abelian symmetry is broken directly at the string level.

The “+” sign was chosen for the hypercharge definition in [8]. In [19], we showed that the choice of the sign in eq. (2.4) has interesting consequences in terms of the decoupling of the exotic fractionally charged states. The other combination of  $U(1)_C$  and  $U(1)_L$ , which is orthogonal to  $U(1)_Y$ , is given by

$$U(1)_{Z'} = U(1)_C \mp U(1)_L. \quad (2.7)$$

In Section 3 we will show that cancellation of the FI term by singlet fields along directions that are  $D$ -flat for all of the non-anomalous  $U(1)$  requires that at least one of the  $U(1)_Y$  and  $U(1)_{Z'}$  be broken. Proof of this is found in Table C.II in Appendix C. Therefore, in the phenomenologically viable case we are forced to have only  $SU(3)_C \times SU(2)_L \times U(1)_Y$  as the unbroken  $SO(10)$  subgroup below the string scale, when only singlets take on VEVs. Under the constraint that only singlets take on VEVs, this is an interesting example of how string dynamics may force the  $SO(10)$  subgroup below the string scale to coincide with the Standard Model gauge group. (Complete proof that only one of  $U(1)_Y$  and  $U(1)_{Z'}$  can survive would necessitate that non-Abelian state VEVs also be considered, which is currently being investigated [24].)

### 2.3 Matter Spectrum and Superpotential

The full massless spectrum of the model, together with the quantum numbers under the right-moving gauge group, were first presented in ref. [8]. In our Tables A.I and A.II of Appendix A, we again list these states and their gauge and global charges. The gauge charges in Table A.I are expressed in the rotated  $U(1)$  basis of eqs. (2.31,2.32) (and of eqs. (18a-f) in [8]), rather than in the unrotated basis associated with the tables of [8].

In the FNY model, the three sectors  $\mathbf{b}_1$ ,  $\mathbf{b}_2$ , and  $\mathbf{b}_3$  correspond to the three twisted sectors of the  $Z_2 \times Z_2$  orbifold model, with each sector producing one generation in the 16 representation,  $(Q_i, u_i^c, d_i^c, L_i, e_i^c, N_i^c)$ , of  $SO(10)$  decomposed under  $SU(3)_C \times SU(2)_L \times U(1)_C \times U(1)_L$ , with charges under the horizontal symmetries. In addition to the gravity and gauge multiplets and several singlets (see below), the untwisted Neveu-Schwarz (NS) sector produces three pairs of electroweak scalar doublets  $\{h_1, h_2, h_3, \bar{h}_1, \bar{h}_2, \bar{h}_3\}$ . Each NS electroweak doublet set  $(h_i, \bar{h}_i)$  may be viewed as a pair of Higgs with the potential to give renormalizable (near EW scale) mass to the corresponding  $\mathbf{b}_i$ -generation of MSSM matter. Thus, to reproduce the MSSM states and generate a viable three generation mass hierarchy, two out of three of these Higgs pairs must become massive near the string/FI scale. The twisted sector provides some additional  $SU(3)_C \times SU(2)_L$  exotics: one  $SU(3)_C$  triplet/anti-triplet pair  $\{H_{33}, H_{40}\}$ ; one  $SU(2)_L$  up-like doublet,  $H_{34}$ , and one down-like doublet,  $H_{41}$ ; and two pairs of vector-like  $SU(2)_L$  doublets,  $\{V_{45}, V_{46}\}$  and  $\{V_{51}, V_{52}\}$ , with fractional electric charges  $Q_e = \pm \frac{1}{2}$ .

The FNY model contains a total of 63 non-Abelian singlets. Three of these are the MSSM electron conjugate states  $(e_1^c, e_2^c, e_3^c)$  and another three are the neutrino



$SU(2)_L$  singlets ( $N_1^c, N_2^c, N_3^c$ ). Of the remaining 57 singlets, 16 possess electric charge and 41 do not. The set of 16 are twisted sector states,\* eight of which carry  $Q_e = \frac{1}{2}$ ,

$$\{H_3^s, H_5^s, H_7^s, H_9^s, V_{41}^s, V_{43}^s, V_{47}^s, V_{49}^s\}, \quad (2.8)$$

and another eight of which carry  $Q_e = -\frac{1}{2}$ ,

$$\{H_4^s, H_6^s, H_8^s, H_{10}^s, V_{42}^s, V_{44}^s, V_{48}^s, V_{50}^s\}. \quad (2.9)$$

Three of the 41  $Q_e = 0$  states,

$$\{\Phi_1, \Phi_2, \Phi_3\}, \quad (2.10)$$

are NS sector singlets of the entire four dimensional gauge group. Another fourteen of these singlets,

$$\{\Phi_{12}, \bar{\Phi}_{12}, \Phi_{23}, \bar{\Phi}_{23}, \Phi_{13}, \bar{\Phi}_{13}, \Phi_{56}, \bar{\Phi}_{56}, \Phi'_{56}, \bar{\Phi}'_{56}, \Phi_4, \bar{\Phi}_4, \Phi'_4, \bar{\Phi}'_4\}, \quad (2.11)$$

form seven pairs,

$$(\Phi_{12}, \bar{\Phi}_{12}), (\Phi_{23}, \bar{\Phi}_{23}), (\Phi_{13}, \bar{\Phi}_{13}), (\Phi_{56}, \bar{\Phi}_{56}), (\Phi'_{56}, \bar{\Phi}'_{56}), (\Phi_4, \bar{\Phi}_4), (\Phi'_4, \bar{\Phi}'_4), \quad (2.12)$$

that are vector-like, i.e., possessing charges of equal magnitude but opposite sign, for all local Abelian symmetries. (Note for later discussion that  $\Phi_4$  and  $\Phi'_4$  carry identical Abelian gauge charges, thereby resulting in six, rather than seven, *distinct* vector-like pairs.) The remaining 24  $Q_e = 0$  singlets,

$$\{H_{15}^s, H_{16}^s, H_{17}^s, H_{18}^s, H_{19}^s, H_{20}^s, H_{21}^s, H_{22}^s, H_{29}^s, H_{30}^s, H_{31}^s, H_{32}^s, H_{36}^s, H_{37}^s, H_{38}^s, H_{39}^s, V_1^s, V_2^s, V_{11}^s, V_{12}^s, V_{21}^s, V_{22}^s, V_{31}^s, V_{32}^s\}, \quad (2.13)$$

are twisted sector states carrying both observable and hidden sector Abelian charges.

The FNY model contains 34 hidden sector non-Abelian states, all of which also carry both observable and hidden  $U(1)_i$  charges: Five of these are  $SU(3)_H$  triplets,

$$\{H_{42}, V_4, V_{14}, V_{24}, V_{34}\}, \quad (2.14)$$

while another five are anti-triplets,

$$\{H_{35}, V_3, V_{13}, V_{23}, V_{33}\}. \quad (2.15)$$

The remaining 24 states include 12  $SU(2)_H$  doublets,

$$\{H_1, H_2, H_{23}, H_{26}, V_5, V_7, V_{15}, V_{17}, V_{25}, V_{27}, V_{39}, V_{40}\} \quad (2.16)$$

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\*Vector-like representations of the hidden sector are denoted by a “V”, while chiral representations are denoted by a “H”. A superscript “s” indicates a non-Abelian singlet.

and a corresponding 12  $SU(2)'_H$  doublets,

$$\{H_{11}, H_{13}, H_{25}, H_{28}, V_9, V_{10}, V_{19}, V_{20}, V_{29}, V_{30}, V_{35}, V_{37}\}. \quad (2.17)$$

The only hidden sector NA states with non-zero  $Q_e$  are the four doublets  $H_1, H_2, H_{11}$ , and  $H_{13}$ , which carry  $Q_e = \pm \frac{1}{2}$ . The sector origins of all exotics is discussed in [8, 16, 19], and a general classification of exotic states in the more realistic free fermionic models is discussed in ref. [25].

For a string derived MSSM to result from the FNY model, the several exotic MSSM-charged states must be eliminated from the low energy effective field theory. Along with two linearly independent combinations of the  $h_{i=1,2,3}$ , and of the  $\bar{h}_{i=1,2,3}$ , the entire set of states,

$$\{H_{33}, H_{40}, H_{34}, H_{41}, V_{45}, V_{46}, V_{51}, V_{52}, \quad (2.18)$$

$$H_1, H_2, H_{11}, H_{12}, \quad (2.19)$$

$$H_3^s, H_5^s, H_7^s, H_9^s, V_{41}^s, V_{43}^s, V_{47}^s, V_{49}^s, \quad (2.20)$$

$$H_4^s, H_6^s, H_8^s, H_{10}^s, V_{42}^s, V_{44}^s, V_{48}^s, V_{50}^s\}. \quad (2.21)$$

must be removed.

Examination of the MSSM-charged state superpotential<sup>†</sup> shows that two out of three of each of the  $h_i$  and  $\bar{h}_i$  Higgs, and *all* of the (2.18–2.21) states, can, indeed, be decoupled from the low energy effective field theory via the terms,

$$\Phi_{12}h_1\bar{h}_2 + \Phi_{23}h_3\bar{h}_2 + H_{31}^s h_2 H_{34} + H_{38}^s \bar{h}_3 H_{41} + \quad (2.22)$$

$$\Phi_4[V_{45}V_{46} + H_1H_2] + \quad (2.23)$$

$$\overline{\Phi}_4[H_3^s H_4^s + H_5^s H_6^s + V_{41}^s V_{42}^s + V_{43}^s V_{44}^s] + \quad (2.24)$$

$$\Phi_4'[V_{51}V_{52} + H_7^s H_8^s + H_9^s H_{10}^s] + \quad (2.25)$$

$$\overline{\Phi}_4'[V_{47}^s V_{48}^s + V_{49}^s V_{50}^s + H_{11}H_{13}] + \quad (2.26)$$

$$\Phi_{23}H_{31}^s H_{38}^s [H_{33}H_{40} + H_{34}H_{41}]. \quad (2.27)$$

This will occur if all states in the set

$$\{\Phi_4, \overline{\Phi}_4, \Phi_4', \overline{\Phi}_4', \Phi_{12}, \Phi_{23}, H_{31}^s, H_{38}^s\} \quad (2.28)$$

take on near string scale VEVs through FI term anomaly cancellation. All but two of the terms in (2.22-2.27) are of third order and will result in unsuppressed FI scale masses, while the remaining two terms are of fifth order. The  $H_{33}H_{40}$  fifth order term may result in  $H_{33}$  and  $H_{40}$  receiving a slightly suppressed mass. On the other hand,

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<sup>†</sup>In Appendix B, we present the FNY superpotential up to sixth order. Terms in the superpotential belong to one of four classes, those containing, in addition to (possible) singlets: (i) nothing else, (ii) only MSSM-charged states, (iii) both MSSM and hidden sector charged-states, and (iv) only hidden sector charged-states.

the  $H_{34}H_{41}$  term has only a minor, perturbative effect on  $H_{34}$  and  $H_{41}$  masses, since  $H_{34}$  and  $H_{41}$  both appear in third order mass terms as well.

In Section 3, we present  $D$ - and  $F$ -flat directions that contain the required fields (2.28) for decoupling all of the SM-charged exotics. Some of these directions are flat to all orders in the superpotential, while others are flat only to finite order. Before discussing these directions though, we review the process by which they were found.

### 2.3.1 Anomalous $U(1)_A$

All known chiral three generation  $SU(3)_C \times SU(2)_L \times U(1)_Y$  models, of lattice, orbifold, or free fermionic construction, contain an anomalous local  $U(1)_A$  [26]. An anomalous  $U(1)_A$  has non-zero trace of its charge over the massless states of the low energy effective field theory,

$$\text{Tr } Q^{(A)} \neq 0 . \quad (2.29)$$

String models often appear to have not just one, but several anomalous Abelian symmetries  $U(1)_{A,i}$  ( $i = 1$  to  $n$ ), each with  $\text{Tr } Q_i^{(A)} \neq 0$ . However, there is always a rotation that places the entire anomaly into a single  $U(1)_A$ , uniquely defined by

$$U(1)_A \equiv c_A \sum_{i=1}^n \{\text{Tr } Q_i^{(A)}\} U(1)_{A,i}, \quad (2.30)$$

with  $c_A$  a normalization coefficient. There are then  $n-1$  traceless  $U(1)'_j$  formed from linear combinations of the  $n$   $U(1)_{A,i}$  and orthogonal to  $U(1)_A$ .

Prior to rotating the anomaly into a single  $U(1)_A$ , six of the FNY model's twelve  $U(1)$  symmetries are anomalous:  $\text{Tr } U_1 = -24$ ,  $\text{Tr } U_2 = -30$ ,  $\text{Tr } U_3 = 18$ ,  $\text{Tr } U_5 = 6$ ,  $\text{Tr } U_6 = 6$  and  $\text{Tr } U_8 = 12$ . Thus, the total anomaly can be rotated into a single  $U(1)_A$  defined by

$$U_A \equiv -4U_1 - 5U_2 + 3U_3 + U_5 + U_6 + 2U_8. \quad (2.31)$$

The five orthogonal linear combinations,

$$\begin{aligned} U'_1 &= 2U_1 - U_2 + U_3 \quad ; \quad U'_2 = -U_1 + 5U_2 + 7U_3 \quad ; \\ U'_3 &= U_5 - U_6 \quad ; \quad U'_4 = U_5 + U_6 - U_8 \\ U'_5 &= 12U_1 + 15U_2 - 9U_3 + 25U_5 + 50U_8 . \end{aligned} \quad (2.32)$$

are all traceless.

## 3 Flat Directions

### 3.1 Constraints on Flat Directions

### 3.1.1 $D$ -constraints

A set of vacuum expectation values (VEVs) will automatically appear in any string model with an anomalous  $U(1)_A$  as a result of the string theory anomaly cancellation mechanism [27]. Following the anomaly rotation of eq. (2.30), the universal Green–Schwarz (GS) relations,

$$\begin{aligned} \frac{1}{k_m k_A^{1/2}} \text{Tr}_{G_m} T(R) Q_A &= \frac{1}{3k_A^{3/2}} \text{Tr} Q_A^3 = \frac{1}{k_i k_A^{1/2}} \text{Tr} Q_i^2 Q_A = \frac{1}{24k_A^{1/2}} \text{Tr} Q_A \\ &\equiv 8\pi^2 \delta_{\text{GS}} , \end{aligned} \quad (3.1)$$

$$\begin{aligned} \frac{1}{k_m k_i^{1/2}} \text{Tr}_{G_m} T(R) Q_i &= \frac{1}{3k_i^{3/2}} \text{Tr} Q_i^3 = \frac{1}{k_A k_i^{1/2}} \text{Tr} Q_A^2 Q_i = \frac{1}{(k_i k_j k_A)^{1/2}} \text{Tr} Q_i Q_{j \neq i} Q_A \\ &= \frac{1}{24k_i^{1/2}} \text{Tr} Q_i = 0 , \end{aligned} \quad (3.2)$$

where  $k_m$  is the level of the non-Abelian gauge group  $G_m$  and  $2T(R)$  is the index of the representation  $R$  of  $G_m$ , defined by

$$\text{Tr} T_a^{(R)} T_b^{(R)} = T(R) \delta_{ab} , \quad (3.3)$$

removes all Abelian triangle anomalies except those involving either one or three  $U_A$  gauge bosons. (The GS relations are a by-product of modular invariance constraints.)

The standard anomaly cancellation mechanism breaks  $U_A$  and, in the process, generates a FI  $D$ -term,

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2} \text{Tr} Q^{(A)} , \quad (3.4)$$

where  $g_s$  is the string coupling and  $M_P$  is the reduced Planck mass,  $M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18}$ .

The form of the FI-term was determined from string theory assumptions. Therefore, a more encompassing  $M$ -theory [28] might suggest modifications to this FI-term. However, recently it was argued that  $M$ -theory does not appear to alter the form of the FI-term [29]. Instead an  $M$ -theory FI-term should remain identical to the FI-term obtained for a weakly-coupled  $E_8 \times E_8$  heterotic string, independent of the size of  $M$ -theory's 11<sup>th</sup> dimension.

Spacetime supersymmetry is broken near the string scale by the FI  $D_A$ -term, unless a set of scalar VEVs,  $\{\langle \varphi_m \rangle\}$ , carrying anomalous charges  $Q_m^{(A)}$  can contribute a compensating  $\langle D_A(\varphi_m) \rangle \equiv \sum_\alpha Q_m^{(A)} |\langle \varphi_m \rangle|^2$  term to cancel the FI-term, i.e.,

$$\langle D_A \rangle = \sum_m Q_m^{(A)} |\langle \varphi_m \rangle|^2 + \epsilon = 0 , \quad (3.5)$$

thereby restoring supersymmetry. The actual set of VEVs accomplishing this cancellation will be dynamically determined by non-perturbative effects. Whichever VEV

combination this may be, the phenomenology of the model will be drastically altered from that which exists before the VEV is applied.

Any set of scalar VEVs satisfying eq. (3.5) must also retain  $D$ -flatness for all of the non-anomalous Abelian  $U_i$  symmetries as well,<sup>‡</sup>

$$\langle D_i \rangle = \sum_m Q_m^{(i)} |\langle \varphi_m \rangle|^2 = 0. \quad (3.6)$$

### 3.1.2 $F$ -constraints

Each superfield  $\Phi_m$  (containing a scalar field  $\varphi_m$  and chiral spin- $\frac{1}{2}$  superpartner  $\psi_m$ ) that appears in the superpotential imposes further constraints on the scalar VEVs.  $F$ -flatness will be broken (thereby destroying spacetime supersymmetry) at the scale of the VEVs unless,

$$\langle F_m \rangle \equiv \left\langle \frac{\partial W}{\partial \Phi_m} \right\rangle = 0; \quad \langle W \rangle = 0. \quad (3.7)$$

$F$ -flatness of a given set of VEVs can be broken by two types of superpotential terms: (i) those composed only of the VEV'd fields, and (ii) those composed, in addition to the VEV'd fields, of a single field without a VEV. (These superpotential terms were called “type A” and “type B” respectively in [30].) Any power of a type A term appearing in a superpotential will break  $F$ . For example if generic fields  $\varphi_1$ ,  $\varphi_2$  and  $\varphi_3$  take on VEVs in a  $D$ -flat direction, then any superpotential term of the form

$$((\Phi_1)^{n_1}(\Phi_2)^{n_2}(\Phi_3)^{n_3})^m \quad (3.8)$$

(with  $\{n_{i=1,2,3}\}$  being a set of relative primes) eliminates  $F$ -flatness at respective order  $m(n_1 + n_2 + n_3)$ . Thus, all order  $F$ -flatness, requires that no terms of this form appear in the superpotential for any combination of  $m$ ,  $n_{i=1,2,3}$  values. On the other hand, when  $\Phi_x$  lacks a VEV, we see from (3.7) that superpotential terms  $((\Phi_1)^{n_1}(\Phi_2)^{n_2}(\Phi_3)^{n_3}(\Phi_x)^{n_x})^m$ , (with  $\{n_{i=1,2,3}, n_x\}$  similarly a set of relative primes)  $F$ -flatness breaking type B terms only when  $n_x = m = 1$ .

Generically, there are many more  $D$ -flat directions that are simultaneously  $F$ -flat to a given order in the superpotential for the effective field theory of a string model than for the field-theoretic counterpart. In particular, there are usually several  $D$ -flat directions that are  $F$ -flat to all order in a string model, but only flat to some finite, often low, order in the corresponding field-theoretic model. This may be attributed to string world-sheet selection rules [31, 32] which impose strong constraints on allowed superpotential terms beyond gauge invariance [22]. For example, for a given set of states, a comparison between the allowed terms in a stringy superpotential and the corresponding gauge invariant field-theoretic superpotential is performed in [33].

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<sup>‡</sup>Here we consider flat directions involving only non-Abelian singlet fields. In cases where non-trivial representations of the non-Abelian gauge groups are also allowed to take on VEVs, generalized non-Abelian  $D$ -flat constraints must also be imposed. See, for example, [24].

### 3.2 $D$ -flat Basis Sets

We continue our flat direction discussion by considering basis sets of  $D$ -flat directions. Let  $\{\varphi_{m=1 \text{ to } n}\}$  denote the  $n$ -dimension set of fields of that are allowed to possibly take on VEVs. Further, let

$$C_j = \{|\langle\varphi_1\rangle_j|^2, |\langle\varphi_2\rangle_j|^2, \dots, |\langle\varphi_n\rangle_j|^2\} \quad (3.9)$$

$$\equiv \{a_{j,1}, a_{j,2}, \dots, a_{j,n}\} , \quad (3.10)$$

with

$$a_{j,x} \equiv |\langle\varphi_x\rangle_j|^2 \geq 0 , \quad (3.11)$$

be a generic set of the norms of VEVs <sup>§</sup> that satisfy all non-anomalous  $D$ -flat constraints eq. (3.6). A set  $C_j$  of such VEVs corresponds to a polynomial of fields  $\varphi_1^{a_{j,1}} \varphi_2^{a_{j,2}} \dots \varphi_n^{a_{j,n}}$ , invariant under all non-anomalous gauge symmetries [34, 35, 30]. A non-anomalous  $D$ -flat direction (3.9) possesses some number of overall scale degrees of freedom (DOFs), which is the dimension of the direction. For example, the norms of the VEVs of a one-dimensional  $D$ -flat direction can be expressed as a product of a single positive real overall scale factor  $\alpha$  and positive semi-definite integral coefficients,  $c_{j,x}^\alpha \geq 0$ , which specify the ratios between the VEVs of the various  $n$  fields,

$$C_j^{1\text{-dim}} = \{c_{j,1}^\alpha \alpha, c_{j,2}^\alpha \alpha, \dots, c_{j,n}^\alpha \alpha\} . \quad (3.12)$$

Similarly, a two-dimensional flat direction involves independent scales  $\alpha$  and  $\beta$ , i.e.,

$$C_j^{2\text{-dim}} = \{c_{j,1}^\alpha \alpha + c_{j,1}^\beta \beta, c_{j,2}^\alpha \alpha + c_{j,2}^\beta \beta, \dots, c_{j,n}^\alpha \alpha + c_{j,n}^\beta \beta\} . \quad (3.13)$$

Any physical  $D$ -flat direction  $C_j$  can be expressed as a linear combination of a set of  $D$ -flat basis directions,  $\{B_1, B_2, B_3, \dots, B_k\}$ :

$$C_j = \sum_k w_{j,k} B_k , \quad (3.14)$$

where  $w_{j,k}$  are real weights. A basis can always be formed in which each element

$$B_k = \{|\langle\varphi_1\rangle_k|^2, |\langle\varphi_2\rangle_k|^2, \dots, |\langle\varphi_n\rangle_k|^2\} \quad (3.15)$$

$$\equiv \{b_{k,1}, b_{k,2}, \dots, b_{k,n}\} , \quad (3.16)$$

where  $b_{k,x} \equiv |\langle\varphi_x\rangle_k|^2$  is an integer, is one-dimensional, i.e.,

$$B_k^{1\text{-dim}} = \{b'_{k,1} \gamma_k, b'_{k,2} \gamma_k, \dots, b'_{k,n} \gamma_k\} , \quad (3.17)$$

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<sup>§</sup>Hereon, we will often, for sake of brevity, refer to the norms of the VEVs simply as the VEVs when it is clear that the norms are implied.

where  $\gamma_k$  is the overall scale factor and  $b'_{k,x}$  are the relative ratios of the VEVs. Further, the scale factor of a basis element can always be normalized to 1, thereby leaving  $B_k$  to be defined solely by the  $b'_{k,x}$ ,

$$B_k^{1-\dim} = \{b'_{k,1}, b'_{k,2}, \dots, b'_{k,n}\} . \quad (3.18)$$

Neither the coefficients  $b'_{k,x}$  [36] of a basis element  $B_k$ , nor the weights  $w_{j,k}$  [30] need all be non-negative, so long as the *total* contribution of all basis elements to an individual norm of a VEV,  $a_{j,x} \equiv \sum_k w_{j,k} b'_{k,x}$ , in a flat direction  $C_j$  is non-negative [30]. However, a basis vector  $B_k$  that contains at least one negative coefficient  $b'_{k,x} < 0$  cannot be viewed as a physical one-dimensional  $D$ -flat direction. Instead it corresponds to a monomial of fields containing at least one field with a negative power,  $\varphi_1^{b'_{k,1}} \varphi_2^{b'_{k,2}} \dots \varphi_n^{b'_{k,n}} / \varphi_x^{|b'_{k,x}|}$ .

Two types of basis sets were primarily discussed in [30]: (i) a basis composed of a maximal set of linearly independent *physical* (i.e., all  $b'_{k,x} \geq 0$ ) one-dimensional  $D$ -flat directions<sup>¶</sup> and (ii) a “superbasis” composed of the set of all (and therefore not all linearly independent) one-dimensional flat directions. The dimension  $d_B$  of a linearly independent physical basis set is less than or equal to  $N_{VEV} - N_d$ , where  $N_{VEV} = n$  is the number of fields allowed VEVs and  $N_d$  is the number of independent non-anomalous  $D$ -constraints from the set of  $N_U$  non-anomalous  $U(1)_i$ .<sup>||</sup> A necessity for saturation of the upper bound for  $d_B$ , is that there is at least one pair of states with charges of opposite sign for each non-anomalous  $U(1)_i$ . When there is some  $U(1)_s$  under which  $N_s$  states all have charges of the same sign and the remaining  $N_{VEV} - N_s$  states are uncharged, none of the  $N_s$  states may take on a VEV and the number of relevant independent  $U(1)_i$  is reduced by one,  $d_B \leq N'_{VEV} - N'_d = (N_{VEV} - N_s) - (N_d - 1) \leq N_{VEV} - N_d$ . In general, determining the effective  $N'_{VEV}$  and  $N'_d$  is an iterative process: After the  $N_s$  states are removed and  $N_d$  decreased by one for  $U(1)_s$ , then we must check again that there are no other  $U(1)_i$  without a pair of states of oppositely sign charges, etc  $\dots$ . Let  $N'_{VEV}$  and  $N'_d$  be the number of states allowed VEVs and number of independent  $D$ -constraints after “eliminating” all  $U(1)_i$  that lack pairs of states with charges of opposite sign. As we discuss in greater detail in the next subsection,  $d_B$  will generally be less than  $N'_{VEV} - N'_d$  because of the very non-trivial positive semi-definite requirement (3.11) on the  $|\langle \phi_k \rangle|^2$  in (3.9,3.10).

There is one possible disadvantage to choosing a basis set of linearly independent physical VEVs to generate higher dimensional physical flat directions  $C_j^{n-\dim}$ . That is, there are usually physical flat directions  $C_j$  that require some negative  $w_{j,k}$  coefficients in (3.14) when  $C_j$  is expressed in terms of physical basis directions. This can complicate the systematic generation of  $D$ -flat directions. On the other hand, using a superbasis of not all linearly independent physical VEVs overcomes this possible difficulty, since all higher dimensional physical  $D$ -flat directions,  $C_i^{n \geq 2-\dim}$ , can be

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<sup>¶</sup>See Table IV of [30] for an FNY non-anomalous flat direction basis of this type.

<sup>||</sup>For a proper subset of states in a model, all  $U(1)_i$  charges may not be independent.

constructed from superbasis elements  $B_k$  using only non-negative coefficients  $|w_{j,k}|$  [30],

$$C_j = \sum_k |w_{j,k}| B_k . \quad (3.19)$$

In practice, however, application of the superbasis approach can, at times, have its own complication: the number of one-dimensional flat directions composing a given superbasis can be extremely large (on the order of several hundred or more) in some string models such as the FNY model. Extremely large dimensions of a superbasis can make systematic generation of  $D$ -flat directions unwieldy.

An alternative to these two types of  $D$ -flat basis sets is what we term the “maximally orthogonal” basis set. This type of basis may, in fact, coincide with a linearly independent physical basis in some models, but more commonly is slightly larger in dimension. In a maximally orthogonal basis, each  $D$ -flat basis element  $B_k^{mo}$  has one non-zero *positive* coefficient  $b_{k,x}^{mo'}$  for which the corresponding  $b_{k',x}^{mo'}$  are zero for all other basis elements  $B_{k' \neq k}^{mo}$ . Each  $B_k^{mo}$  becomes associated with a particular field  $\varphi_x$ . Hence, all  $w_{j,k}$  defining a physical non-anomalous  $D$ -flat direction  $C_j$  must be non-negative since

$$a_{j,x} \equiv |\langle \varphi_x \rangle_j|^2 = w_{j,k} b_{k,x}^{mo'} \geq 0 . \quad (3.20)$$

Otherwise a flat direction involving a negative  $w_{j,k}$  weight would have at least one VEV with a negative norm and would, therefore, be unphysical.

Associating one component  $b_{k,x}^{mo'}$  of a maximally orthogonal basis element  $B_k^{mo}$  with the field  $\varphi_x$  often results in a few of the other components  $b_{k,x' \neq x}^{mo'}$  being negative. Hence, several  $B_k^{mo}$  may correspond to unphysical directions. This does not present any real difficulty though. Rather, production of only physical directions  $C_j$  then simply places some constraints on linear combinations of the  $B_k$ . A “maximally orthogonal” basis set of flat directions has essentially the same advantage as a superbasis, yet can keep the dimension of the basis set reasonable when the dimension of the superbasis may be unfeasibly large.

There is one case in which the positivity constraint (3.11) discussed above is effectively relaxed. The exception to (3.11) occurs when two states can be combined into vector-like pairs. Let us denote a generic pair as  $\varphi_m$  and  $\varphi_{-m}$ . For this pair,  $a_{j,m}$  in (3.10) may take on negative values in a physical flat direction, because a vector-like pair of states acts effectively like a single state with regard to all  $D$ -constraints. For a flat direction,  $C_j$ , the contribution of this pair to each  $D$ -term is

$$\langle D_i \rangle (\varphi_m, \varphi_{-m}) = Q_m^{(i)} |\langle \varphi_m \rangle_j|^2 + Q_{-m}^{(i)} |\langle \varphi_{-m} \rangle_j|^2 \quad (3.21)$$

$$\equiv Q_m^{(i)} a_{j,m} + Q_{-m}^{(i)} a_{j,-m} . \quad (3.22)$$

Since by definition  $Q_{-m}^{(i)} = -Q_m^{(i)}$ , eq. (3.22) can be rewritten as

$$\langle D_i \rangle (\varphi_m, \varphi_{-m}) = Q_m^{(i)} (a_{j,m} - a_{j,-m}) \quad (3.23)$$

$$\equiv Q_m^{(i)} a'_{j,m} , \quad (3.24)$$



where  $a'_{j,m} \equiv (a_{j,m} - a_{j,-m})$  may be positive, negative, or zero.

We can consider  $a'_{j,m}$  as originating from a single field  $\langle \varphi'_m \rangle$ . This allows us to reduce the effective number of nontrivial states,  $N_{VEV}$ , by one for each vector pair [10, 36]. This reduction is compensated by an additional *trivial*  $D$ -flat direction basis element,  $\langle \varphi_m \rangle = \langle \varphi_{-m} \rangle$ , (corresponding to the binomial  $\varphi_m \varphi_{-m}$ ) formed from both components of a vector pair. The FNY model contains several vector-like pairs and provides an excellent example of reduction of effective nontrivial states.

Having all states in vector-like pairs is equivalent to totally relaxing the “positivity” constraint. As a general rule, the more vector pairs of non-Abelian singlets there are, the more likely a  $D$ -flat FI-term cancelling direction can be formed.

### 3.2.1 Maximally Orthogonal Basis Sets Via Singular Value Decomposition

One method for generating a maximally orthogonal basis set of  $D$ -flat directions for the non-anomalous  $U(1)_i$  involves singular value decomposition (SVD) of a matrix [37]. While the matrix [36] method and the more standard monomial approach [34, 35, 30] are essentially different languages for the same process, a strength of the matrix decomposition method is that it generates a complete basis of  $D$ -flat directions for non-Abelian singlet states *en masse*. We briefly discuss the SVD approach here, since it provides a somewhat new interpretation to flat directions.

SVD is based on the mathematical fact that any  $(M \times N)$ -dimensional matrix  $\mathbf{D}$  whose number of rows  $M$  is greater than or equal to its number of columns  $N$ , can be written as the product of an  $M \times N$  column-orthogonal matrix  $\mathbf{U}$ , an  $N \times N$  diagonal matrix  $\mathbf{W}$  containing only semi-positive-definite elements, and the transpose of an  $N \times N$  orthogonal matrix  $\mathbf{V}$  [37],

$$\mathbf{D}_{M \times N} = \mathbf{U}_{M \times N} \cdot \mathbf{W}_{N \times N}^{\text{diag}} \cdot \mathbf{V}_{N \times N}^T, \quad \text{for } M \geq N. \quad (3.25)$$

This decomposition is always possible, no matter how singular the matrix is. The decomposition is also nearly unique, up to (i) making the same permutation of the columns of  $U$ , diagonal elements of  $W$ , and columns of  $V$ , or (ii) forming linear combinations of any columns of  $U$  and  $V$  whose corresponding elements of  $W$  are degenerate. If initially  $M < N$ , then a  $(N - M) \times N$  zero-matrix can always be appended onto  $\mathbf{D}$  so this decomposition can be performed:  $\mathbf{D}_{(M < N), N} \rightarrow \mathbf{D}'_{(M=N), N}$ .

SVD is extremely useful when the matrix  $\mathbf{D}^{**}$  is associated with a set of  $M$  simultaneous linear equations expressed by,

$$\mathbf{D} \cdot \vec{x} = \vec{b}, \quad (3.26)$$

where  $x$  and  $b$  are vectors. Eq. (3.26) defines a linear mapping from  $N$ -dimensional vector space  $x$  to  $M$ -dimensional vector-space  $b$ .

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\*\*We assume from hereon that  $\mathbf{D}$  has been enhanced by a zero submatrix if necessary so that  $M \geq N$ .

For  $M = N$ ,  $\mathbf{D}$  is singular when at least one of the  $M$  constraints is not linearly independent. Associated with a singular  $\mathbf{D}$  is a subspace of  $\vec{x}$  termed the *nullspace*,  $\mathcal{M}_{null}$ , that is mapped to  $\vec{0}$  in  $b$ -space by  $\mathbf{D}$ . The dimension of this nullspace is referred to as the *nullity*. The subspace of  $\vec{b}$  that *can* be reached by the matrix  $\mathbf{D}$  acting on  $\vec{x}$  is called the *range* of  $\mathbf{D}$ . The dimension of the range is denoted as the *rank* of  $\mathbf{D}$  and is equal to the number of independent constraint equations  $\equiv M' \leq M$ . Clearly

$$\text{rank } \mathbf{D} + \text{nullity } \mathbf{D} = N. \quad (3.27)$$

In the decomposition of  $\mathbf{D}$  in (3.25), the columns of  $\mathbf{U}$  corresponding to the non-zero diagonal components of  $\mathbf{W}$  form an orthonormal set of basis vectors that span the range of  $D$ . Alternately, the columns of  $\mathbf{V}$  corresponding to the zero diagonal components of  $\mathbf{W}$  form an orthonormal basis for the nullspace.

As is perhaps obvious, this method is directly applicable to constructing  $D$ -flat directions, especially when only non-Abelian singlet states are allowed VEVs. Let  $M = N_U$  ( $M_I = N_d$ ) denote the number of (independent)  $D$ -flat constraints and  $N = N_{VEV}$ , the number of fields allowed to take on VEVs. Then the  $D_{i,j}$  component of the matrix  $\mathbf{D}$  is the  $Q_j^{(i)}$  charge of the state  $\varphi_j$ . ( $i$  takes on the value  $A$  for the anomalous  $U(1)_A$  and values  $\{a = 1 \text{ to } M - 1\}$  for the set of non-anomalous  $U(1)_{i.}$ ) The components of the vector  $x$  are the values of  $|\langle \phi_j \rangle|^2$ , and  $b$  has all zero-components except in its  $U(1)_A$  position. The value of  $b$  in the anomalous position is  $-\epsilon$  from eq. (3.5).

Let  $\mathbf{D}'$  be the matrix that excludes the row of anomalous charges in  $\mathbf{D}$ . In this language, the dimension  $d_B$  of the moduli space of flat directions (not necessarily all physical) for the  $M' \equiv M - 1 = N_U - 1$  non-anomalous  $U(1)_{i=1 \text{ to } M'}$  is the nullity of matrix  $\mathbf{D}'$ , denoted as  $\dim \mathcal{M}'_{null}$ , formed from the  $M'$  non-anomalous  $D$ -flat constraints. In other words, the nullity of  $\mathbf{D}'$  is all VEVs formed from combinations of states that have zero net charge in each non-anomalous direction.  $\dim \mathcal{M}'_{null}$  is in the range

$$N - M' \leq \dim \mathcal{M}'_{null} = N - M'_I \leq N, \quad (3.28)$$

where  $N$  is the number of states allowed VEVs and  $M'_I$  is the number of independent non-anomalous constraints.

For the matrix  $\mathbf{D}$ , which contains the anomalous charges, the elements of the range corresponding to anomaly cancelling  $D$ -flat directions are those formed solely from linear combinations of elements of the nullity of  $\mathbf{D}'$  that generate an anomalous component for  $\vec{b}$  of opposite sign to the FI term,  $\epsilon$ . The nullspace of  $\mathbf{D}$  will likewise be formed from linear combinations of  $\mathbf{D}'$ 's nullity elements that generate a zero anomalous component for  $\vec{b}$ . The dimension of the  $\mathbf{D}'$  nullity subset that projects into the range of  $\mathbf{D}$ , denoted by  $\dim \mathcal{M}'_R$ , is 1 (since the anomalous constraint must necessarily be independent of the non-anomalous constraints). An  $(N - M'_I - 1)$ -dimensional subset of  $\mathcal{M}'_{null}$  forms the nullity of  $\mathbf{D}$ .

Our maximally orthogonal basis set for FNY was obtained using the SVD routine in [37]. After eliminating the row of the  $\mathbf{D}_{(10+23)\times 33}^{fny}$  charge matrix corresponding to the anomalous  $U(1)_A$   $D$ -constraint, singular value decomposition was performed on the reduced matrix  $\mathbf{D}_{9\times 33}^{fny'}$ ,

$$\mathbf{D}_{(9+24)\times 33}^{fny'} = \mathbf{U}_{(9+24)\times 33}^{fny'} \cdot \mathbf{W}_{33\times 33}^{fny'} \cdot \mathbf{V}_{33\times 33}^{fny'T} . \quad (3.29)$$

As discussed in the following subsections, 33 is the number of FNY nontrivial singlets allowed VEVs and 9 is the number of effective  $D$ -constraints. An initial basis set of  $D$ -flat directions of dimension  $d_b = 24 = \dim \mathcal{M}'_{null} = 33 - 9$ , was obtained. The components of these 24 basis directions are the components of the 24 columns of  $\mathbf{V}$  for which the diagonal components of  $\mathbf{W}$  is zero. While these 24 basis directions were not initially in maximally orthogonal form, a simple rotation transformed them into this.

### 3.3 FNY Flat Directions

#### 3.3.1 $D$ -flat Basis

The FNY model contains a total of 63 non-Abelian singlets. These fields, along with their local  $U(1)$  and global world-sheet charges, are listed in Appendix A. Of the 63 singlets, 14 can be used to form seven pairs of vector-like singlets:  $(\Phi_{12}, \bar{\Phi}_{12})$ ,  $(\Phi_{13}, \bar{\Phi}_{13})$ ,  $(\Phi_{23}, \bar{\Phi}_{23})$ ,  $(\Phi_{56}, \bar{\Phi}_{56})$ ,  $(\Phi'_{56}, \bar{\Phi}'_{56})$ ,  $(\Phi_4, \bar{\Phi}_4)$ ,  $(\Phi'_4, \bar{\Phi}'_4)$ . The two  $\Phi_4$ -related pairs possess identical gauge charges, so we will refer to the model as having six *distinct* vector-like pairs of singlets and 49 non-vector-like singlets.

Since we wish the SM gauge group to survive after cancellation of the FI term, we investigate flat directions involving only singlets not carrying hypercharge. The set of hypercharge-free singlets is composed of the six distinct vector-like pairs and 27 non-vectors,  $H_{15 \text{ to } 22}^s$ ,  $H_{29 \text{ to } 32}^s$ ,  $H_{36 \text{ to } 39}^s$ ,  $V_{1 \text{ to } 2}^s$ ,  $V_{11 \text{ to } 12}^s$ ,  $V_{21 \text{ to } 22}^s$ ,  $V_{31 \text{ to } 32}^s$ , and  $N_{1 \text{ to } 3}^c$ . These 33 vector-like and non-vector-like singlets carry varying combinations of  $U(1)_A$  and nine other  $U(1)_i$  charges.<sup>††</sup> Hence, from these singlets we can form  $33 - 9 = 24$  non-trivial  $D$ -flat basis directions (some physical, others not physical).

A non-trivial basis set may be generated by several methods, which can give differing properties to the set. As discussed in the preceding subsection, we chose the singular value decomposition method to generate a maximally orthogonal basis set, for which there is a one-to-one correspondence between the 24 flat directions in the basis set and 24 of the 33 distinct states allowed VEVs. Eight more *trivial* basis elements are formed from vector-like pairs. There are eight rather than six vector-like pairs because of the gauge charge redundancy between the two vector-like pairs,  $(\Phi_4, \bar{\Phi}_4)$  and  $(\Phi'_4, \bar{\Phi}'_4)$ .

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<sup>††</sup>While there are ten non-anomalous  $U(1)_i$  besides hypercharge, all singlets charged under  $U(1)_9$  of the hidden sector also carry non-zero hypercharge. Thus, none of the singlets we allow to take on a VEV carry  $U(1)_9$  charge.

In Table C.I, the first six components of each basis element correspond to the norms of the VEVs of the six distinct vector-like pairs. The norms of the VEVs of these fields may be either positive or negative in a physical flat direction. A positive norm implies a field  $\phi_m$  takes on the VEV, while a negative norm implies the vector partner field  $\phi_{-m}$  of opposite charge takes on the VEV. The remaining 27 components of each basis element give the norms of the non-vector-like singlet fields.

A true physical flat direction formed from a combination of basis elements of  $D$ -flat directions must have positive semi-definite norms for all of its non-vector-like fields. However, as discussed earlier, corresponding components of the basis elements need not. A basis element with a negative norm of a non-vector field simply is not a physical flat direction. Our maximally orthogonal method associates as many basis directions with non-vector fields as possible. In the FNY model, this is possible to do for 23 of the 24 basis directions. Thus, for the FNY model, a given basis direction could have up to  $33 - 23 - 6 = 4$  VEV non-vector-like negative component norms, any number of which would imply a flat direction is “non-physical.” The four non-vector-like fields with potentially negative norms are  $N_1^c$ ,  $H_{39}^s$ ,  $H_{16}^s$ , and  $N_3^c$ .<sup>‡‡</sup> Thus, the physical constraints,

$$\sum_{k=1}^{24} w_k b_{k,N_1^c}^{mo'} \geq 0 ; \quad \sum_{k=1}^{24} w_k b_{k,H_{39}^s}^{mo'} \geq 0 ; \quad \sum_{k=1}^{24} w_k b_{k,H_{16}^s}^{mo'} \geq 0 ; \quad \sum_{k=1}^{24} w_k b_{k,N_3^c}^{mo'} \geq 0 , \quad (3.30)$$

must be imposed upon the weight factors  $w_k$  in (3.14) through which physical flat directions  $C_j$  are formed from the basis elements  $B_k^{mo}$ .

An additional constraint on physical flat directions is, of course, that the net anomalous charge  $Q^{(A)}$  must be negative,

$$Q^{(A)}(C_j) = \sum_{k=1}^{24} w_k Q^{(A)}(B_k) < 0 , \quad (3.31)$$

since  $\epsilon > 0$  in eq. (3.5). Of the 24 non-trivial basis directions, nine carry negative anomalous charge, while eight carry positive anomalous charge and seven do not carry this charge.

The SVD approach was also used to generate a set of basis directions that simultaneously conserve both  $U(1)_Y$  and  $U(1)_{Z'}$ . The elements of this five-dimensional set are presented in Table C.II. Interestingly, none of these basis directions have negative anomalous charge: all are, in fact, chargeless under  $U(1)_A$ ! Thus, FI-term cancelling  $D$ -flat directions can never be formed by non-Abelian singlets if both  $U(1)_Y$  and  $U(1)_{Z'}$  are to survive. In other words, singlet flat directions imply the reduction of observable  $SO(10)$  to exactly the SM,  $SU(3)_C \times SU(2)_L \times U(1)_Y$ .

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<sup>‡‡</sup>The VEVs of these four fields appear as the last components of the basis directions in Table C.I of Appendix C.

### 3.3.2 $F$ -flat Directions

In Table D.I of Appendix D, we present several classes of directions in the parameter space of VEVs that are flat up to at least sixth order in the FNY superpotential. These classes are defined solely by the states that take on VEVs, rather than the specific ratios of the VEVs. The first three classes of directions are flat to *all* order in the superpotential. The fourth class is broken by two twelfth order type A terms, denoted as “12–1” and “12–2” in Table D.II, while the fifth through eighth classes are all broken at seventh order by a single term, denoted as “7–1” in Table D.II. Numerous (specifically 22) classes are broken at sixth order, again all by a single term, “6–1.” Three of the sixth order classes are simultaneously broken by an additional sixth order term, “6–2.” Thus, it appears likely that FNY  $D$ -flat directions producing the MSSM spectrum are either  $F$ -flat to all finite orders or experience  $F$ -breaking at twelfth order or lower.

Our MSSM flat directions were found via a computer search that generated combinations of the maximally orthogonal  $D$ -flat basis directions  $B_k^{mo}$  given in Table C.I of Appendix C. Linear combinations of up to nine  $L_j$ -class basis directions were surveyed, with the range of the non-zero integer weights,  $w_k$ , in (3.14) being from one to ten. To eliminate redundancy of flat directions, the set of non-zero  $w_k$  in a given linear combination was required to be relatively prime, i.e., the greatest common factor among any permitted set of  $w_k$  was 1. The combinations of basis directions producing our 30 classes of flat directions is given in Table D.IV. The examples in each of the 30 flat direction classes were formed from five, six, or seven  $L_j$ -class basis directions. We found that all  $D$ -flat directions involving eight or more maximally orthogonal basis directions experienced breaking of  $F$ -flatness at fifth order or lower.

In our computer search, we required flat directions to minimally contain VEVs for the set of states,

$$\{\Phi_4, \bar{\Phi}_4, \Phi'_4, \bar{\Phi}'_4, \Phi_{12}, \Phi_{23}, H_{31}^s, H_{38}^s\}, \quad (3.32)$$

necessary for decoupling of all 32 SM-charged MSSM exotics, comprised of the four extra Higgs doublets and the 28 exotics identified in (2.18–2.21). Imposing this, we found VEVs of  $H_{15}^s$  and  $H_{30}^s$  were also always present. We refer to the set of VEVs of these ten fields in Appendix D as “ $\{VEV_1\}$ .” All of the eight directions broken at seventh order or higher additionally contained the VEV of  $\bar{\Phi}_{56}$ . The three classes of directions  $F$ -flat to all finite order also involved (i) no other VEVs, (ii) the VEV of  $\bar{\Phi}'_{56}$ , and (iii) the VEV of  $H_{19}^s$ , respectively. The class broken at twelfth order additionally included the VEV of  $H_{20}^s$ . Note that the trilinear superpotential term  $\bar{\Phi}'_{56} H_{19}^s H_{20}^s$  allows only one of  $\bar{\Phi}'_{56}$ ,  $H_{19}^s$ , and  $H_{20}^s$  to receive a VEV in any flat direction.

In all directions with seventh order flatness, the sneutrino  $SU(2)_L$  singlet  $N_1^c$  takes on a VEV, as do one of  $\bar{\Phi}'_{56}$ ,  $H_{19}^s$ , or  $H_{20}^s$  and/or  $V_{31}^s$ . For the 22 sixth order classes, subsets of  $\{N_3^c, H_{17}^s, H_{18}^s, H_{21}^s, H_{39}^s, V_{12}^s\}$  obtain VEVs, along with various combinations from  $\{\bar{\Phi}_{56}, \Phi'_{56}, H_{19}^s, H_{20}^s, V_{31}^s, N_1^c\}$ .

Systematic generation of flat directions of the FNY model is efficiently performed using a maximally orthogonal basis. However, the dimension (i.e., the number of VEV scale degrees of freedom)  $Dim$  of a given direction is not always apparent from this approach. To determine  $Dim$ , we also express each flat direction class in terms of its embedded *physical* one-dimensional  $D$ -flat directions. (See Tables D.IV and D.V.) Prior to FI term cancellation, the dimension of a given MSSM flat direction equals the number of embedded physical dimension-one  $D$ -flat directions. Cancellation of the FI term removes one degree of freedom, so the dimension after FI term cancellation,  $Dim_{FI}$ , is one less than  $Dim$ .

The number  $N_B$  of non-anomalous  $U(1)_i$  broken along a given direction is the difference between the number of independent VEVs,  $N_{VEV}$ , and the dimension  $Dim$ ,

$$N_{VEV} - Dim = N_B. \quad (3.33)$$

Or, equivalently,

$$N_{VEV} - Dim_{FI} = N_B + 1. \quad (3.34)$$

As Table D.VI shows, the all-order and twelfth order  $F$ -flat directions break seven non-anomalous  $U(1)_i$ , while the seventh order flat directions break eight. The sixth order directions remove anywhere from eight to ten non-anomalous  $U(1)_i$ .

A pair of physical one-dimensional flat directions, denoted “ $X$ ” and “ $Y$ ”, are at the heart of 27 (out of 30) of our MSSM directions.  $X$  is identified with the class 1 all-order flat direction\* and is the root of the three all-order, the one twelfth order, the four seventh order, and five of the sixth order flat directions. The seventh and higher order directions contain anywhere from zero to three additional physical dimension-one directions. At the root of 14 sixth order directions is the direction  $Y$ .  $X$  and  $Y$  are simultaneously embedded in five of the sixth order directions, while three of the sixth order directions contain neither.

Our MSSM flat direction classes have a property not common to generic stringy flat directions. Specifically,  $F$ -flatness of the MSSM directions is broken by the stringy superpotential at exactly the same level as it would be in a field-theoretic gauge invariant superpotential. That is, stringy world sheet constraints do not remove all of the lowest order dangerous gauge-invariant terms. The equivalent string and gauge-invariant seventh and twelfth order  $F$ -breaking essentially results from all of the  $\Phi_4$ ,  $\Phi'_4$ ,  $\overline{\Phi}_4$ , and  $\overline{\Phi}'_4$  states necessarily taking on VEVs.

## 4 Discussion

We have investigated  $D$ - and  $F$ -flat directions in the FNY model of [8, 16]. The FNY model possesses two aspects generic to many classes of three family  $SU(3)_C \times$

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\*The class 1 flat direction was first formed in [30] from the combination of VEVs denoted  $M_6$ ,  $M_7$  and  $R_{10}$ . Table V of [30] also contains several FNY  $Dim_{FI} = 0$  non-MSSM flat directions.

$SU(2)_L \times U(1)_Y$  string models: both an extra local anomalous  $U(1)_A$  and numerous (often fractionally charged) exotic particles beyond the MSSM. We found several flat directions involving only non-Abelian singlet fields that near the string scale can simultaneously break the anomalous  $U(1)_A$  and give mass to *all* exotic SM-charged observable particles, decoupling them from the low energy spectrum. We were thus able to produce the first known examples of Minimal Superstring Standard Models. Some of our flat directions were shown to be flat to *all* finite orders in the superpotential.

The models produced by our flat directions are consistent with, and may in fact offer the first potential realizations of, the recent conjecture by Witten of possible equivalence between the string scale and the minimal supersymmetric standard model unification scale  $M_U \approx 2.5 \times 10^{16}$  GeV. This conjecture indeed suggests that the observable gauge group just below the string scale should be  $SU(3)_C \times SU(2)_L \times U(1)_Y$  and that the  $SU(3)_C \times SU(2)_L \times U(1)_Y$ -charged spectrum of the observable sector should consist solely of the MSSM spectrum.

We have also discovered that the FNY model provides an interesting example of how string dynamics may force the  $SO(10)$  subgroup below the string scale to coincide with the SM gauge group. When only non-Abelian singlets take on VEVs, we have shown that  $U(1)_Y$  or  $U(1)_{Z'}$  of  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Z'} \in SO(10)$  is necessarily broken. Reversing the roles of  $U(1)_Y$  and  $U(1)_{Z'}$  corresponds to flipping the components in each  $SU(2)_L$  doublet.

The phenomenology obtained from our various flat directions will be studied in [20]. In particular, we will examine the mass hierarchies of the three generations of SM quarks and leptons, the hidden sector dynamics, and issues such as proton decay. For each flat direction, we will present the resulting superpotential after decoupling of the states turned massive via FI-term cancelling VEVs. The rich space of flat directions that we found in the present paper suggests the exciting prospect that one of these flat directions may accommodate all of the phenomenological constraints imposed by the Supersymmetric Standard Model phenomenology. Furthermore, the rich space of solutions may be even further enlarged by adding VEVs of the non-Abelian fields. It ought to be emphasized that it is the promising structure, afforded by the NAHE set, which enables this promising scenario. To highlight this important fact, NAHE-based models should be contrasted with the non-NAHE based models, which although having three generations with the Standard Model gauge group, do not allow the standard  $SO(10)$  embedding of the Standard Model spectrum and contain massless exotic states that cannot be decoupled. Thus, we emphasize once again, that although suggesting a specific three generation model as the true string vacuum, seems still premature, the concrete results, obtained in the analysis of specific models, highlight the underlying, phenomenologically successful, structure generated by the NAHE set. Therefore, it suggests that the true string vacuum could be in the vicinity of these models. That is, it is a  $Z_2 \times Z_2$  model in the vicinity of the free fermionic point in the Narain moduli space, also containing several, perhaps still unknown, additional

Wilson lines. Such Wilson lines correspond in the fermionic language to the boundary condition basis vectors beyond the NAHE set.

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## A String Quantum Number of All FNY Massless Fields

State	$U_E$	$(C, L)_Y$	$U_A$	$U_C$	$U_L$	$U'_1$	$U'_2$	$U'_3$	$U'_4$	$U'_5$	$U_4$	$(3, 2, 2')_H$	$U_7$	$U_H$	$U_9$
$Q_1$	$\frac{2,-1}{3}$	$(3, 2)_{\frac{1}{6}}$	8	2	0	-4	2	0	0	-24	2	(1,1,1)	0	0	0
$Q_2$	$\frac{2,-1}{3}$	$(3, 2)_{\frac{1}{6}}$	12	2	0	2	-10	2	2	20	0	(1,1,1)	0	0	0
$Q_3$	$\frac{2,-1}{3}$	$(3, 2)_{\frac{1}{6}}$	8	2	0	2	14	-2	2	32	0	(1,1,1)	0	0	0
$d_1^c$	$\frac{1}{3}$	$(\bar{3}, 1)_{\frac{1}{3}}$	8	-2	4	-4	2	0	0	-24	-2	(1,1,1)	0	0	0
$d_2^c$	$\frac{1}{3}$	$(\bar{3}, 1)_{\frac{1}{3}}$	8	-2	4	2	-10	-2	-2	-80	0	(1,1,1)	0	0	0
$d_3^c$	$\frac{1}{3}$	$(\bar{3}, 1)_{\frac{1}{3}}$	4	-2	4	2	14	2	-2	-68	0	(1,1,1)	0	0	0
$u_1^c$	$-\frac{2}{3}$	$(\bar{3}, 1)_{-\frac{2}{3}}$	8	-2	-4	-4	2	0	0	-24	-2	(1,1,1)	0	0	0
$u_2^c$	$-\frac{2}{3}$	$(\bar{3}, 1)_{-\frac{2}{3}}$	12	-2	-4	2	-10	2	2	20	0	(1,1,1)	0	0	0
$u_3^c$	$-\frac{2}{3}$	$(\bar{3}, 1)_{-\frac{2}{3}}$	8	-2	-4	2	14	-2	2	32	0	(1,1,1)	0	0	0
$H_{33}$	$-\frac{1}{3}$	$(3, 1)_{-\frac{1}{3}}$	8	-1	-2	-2	-11	2	-4	32	0	(1,1,1)	-1	3	0
$H_{40}$	$\frac{1}{3}$	$(\bar{3}, 1)_{\frac{1}{3}}$	0	1	2	2	-13	-2	-4	56	0	(1,1,1)	1	-3	0
$L_1$	0,-1	$(1, 2)_{-\frac{1}{2}}$	8	-6	0	-4	2	0	0	-24	2	(1,1,1)	0	0	0
$L_2$	0,-1	$(1, 2)_{-\frac{1}{2}}$	8	-6	0	2	-10	-2	-2	-80	0	(1,1,1)	0	0	0
$L_3$	0,-1	$(1, 2)_{-\frac{1}{2}}$	4	-6	0	2	14	2	-2	-68	0	(1,1,1)	0	0	0
$h_1$	0,-1	$(1, 2)_{-\frac{1}{2}}$	16	0	-4	-8	4	0	0	-48	0	(1,1,1)	0	0	0
$h_2$	0,-1	$(1, 2)_{-\frac{1}{2}}$	-20	0	-4	-4	20	0	0	60	0	(1,1,1)	0	0	0
$h_3$	0,-1	$(1, 2)_{-\frac{1}{2}}$	-12	0	-4	-4	-28	0	0	36	0	(1,1,1)	0	0	0
$\bar{h}_1$	1, 0	$(1, 2)_{\frac{1}{2}}$	-16	0	4	8	-4	0	0	48	0	(1,1,1)	0	0	0
$\bar{h}_2$	1, 0	$(1, 2)_{\frac{1}{2}}$	20	0	4	4	-20	0	0	-60	0	(1,1,1)	0	0	0
$\bar{h}_3$	1, 0	$(1, 2)_{\frac{1}{2}}$	12	0	4	4	28	0	0	-36	0	(1,1,1)	0	0	0
$H_{34}$	1,0	$(1, 2)_{\frac{1}{2}}$	8	3	2	-2	-11	2	-4	32	0	(1,1,1)	-1	3	0
$H_{41}$	0,-1	$(1, 2)_{-\frac{1}{2}}$	0	-3	-2	2	-13	-2	-4	56	0	(1,1,1)	1	-3	0
$V_{45}$	$\pm\frac{1}{2}$	$(1, 2)_0$	12	0	0	2	-10	-2	2	20	-2	(1,1,1)	2	0	-2
$V_{46}$	$\pm\frac{1}{2}$	$(1, 2)_0$	-12	0	0	-2	10	2	-2	-20	-2	(1,1,1)	-2	0	2
$V_{51}$	$\pm\frac{1}{2}$	$(1, 2)_0$	-4	0	0	-2	-14	2	2	68	-2	(1,1,1)	-2	0	2
$V_{52}$	$\pm\frac{1}{2}$	$(1, 2)_0$	4	0	0	2	14	-2	-2	-68	-2	(1,1,1)	2	0	-2
$\text{tr } Q_o$			392	-36	0	0	-36	0	-24	-168	-12		0	0	0

Table A.I.a: Gauge Charges of FNY Observable Sector  $SU(3)_C \times SU(2)_L \times U(1)_Y$  Non-Abelian (NA) States. (Charges of both electron conjugates  $e^c$  and neutrino singlets  $N^c$  appear in Table A.I.b with non-Abelian singlets.) The names of the states appear in the first column, with the states' various charges appearing in the other columns. The entries under  $(C, L)_Y$  denote Standard Model charges, while the entries under  $(3, 2, 2')$  denote hidden sector  $SU(3)_H \times SU(2)_H \times SU(2)'_H$  charges. The entries in the last row give the traces of the  $U(1)_i$  over these states. (Note that

all Table A.I  $U_A$  through  $U_9$  charges have been multiplied by a factor of 4 compared to those charges given in ref. [8].)

State	$U_E$	$(C, L)_Y$	$U_A$	$U_C$	$U_L$	$U'_1$	$U'_2$	$U'_3$	$U'_4$	$U'_5$	$U_4$	$(3, 2, 2')_H$	$U_7$	$U_H$	$U_9$
$e_1^c$	1	$(1, 1)_1$	8	6	4	-4	2	0	0	-24	-2	$(1, 1, 1)$	0	0	0
$e_2^c$	1	$(1, 1)_1$	12	6	4	2	-10	2	2	20	0	$(1, 1, 1)$	0	0	0
$e_3^c$	1	$(1, 1)_1$	8	6	4	2	14	-2	2	32	0	$(1, 1, 1)$	0	0	0
$H_3^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-8	3	2	2	11	2	4	-32	2	$(1, 1, 1)$	-1	-3	2
$H_4^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	8	-3	-2	-2	-11	-2	-4	32	2	$(1, 1, 1)$	1	3	-2
$H_5^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-4	3	2	2	11	-2	-4	68	2	$(1, 1, 1)$	-1	-3	-2
$H_6^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	4	-3	-2	-2	-11	2	4	-68	2	$(1, 1, 1)$	1	3	2
$H_7^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	4	3	2	2	-13	-2	0	156	-2	$(1, 1, 1)$	-1	-3	2
$H_8^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	-4	-3	-2	-2	13	2	0	-156	-2	$(1, 1, 1)$	1	3	-2
$H_9^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-8	3	2	2	-13	2	0	-144	-2	$(1, 1, 1)$	-1	-3	-2
$H_{10}^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	8	-3	-2	-2	13	-2	0	144	-2	$(1, 1, 1)$	1	3	2
$V_{41}^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-12	0	4	-2	10	2	-2	-20	2	$(1, 1, 1)$	2	0	-2
$V_{42}^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	12	0	-4	2	-10	-2	2	20	2	$(1, 1, 1)$	-2	0	2
$V_{43}^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	8	0	4	2	-10	2	-2	-80	2	$(1, 1, 1)$	-2	0	2
$V_{44}^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	-8	0	-4	-2	10	-2	2	80	2	$(1, 1, 1)$	2	0	-2
$V_{47}^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	8	0	4	2	14	2	2	32	2	$(1, 1, 1)$	-2	0	2
$V_{48}^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	-8	0	-4	-2	-14	-2	-2	-32	2	$(1, 1, 1)$	2	0	-2
$V_{49}^s$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-4	0	4	-2	-14	2	2	68	2	$(1, 1, 1)$	2	0	-2
$V_{50}^s$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	4	0	-4	2	14	-2	-2	-68	2	$(1, 1, 1)$	-2	0	2
$N_1^c$	0	$(1, 1)_0$	8	6	-4	-4	2	0	0	-24	-2	$(1, 1, 1)$	0	0	0
$N_2^c$	0	$(1, 1)_0$	8	6	-4	2	-10	-2	-2	-80	0	$(1, 1, 1)$	0	0	0
$N_3^c$	0	$(1, 1)_0$	4	6	-4	2	14	2	-2	-68	0	$(1, 1, 1)$	0	0	0
$\Phi_1$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	0	$(1, 1, 1)$	0	0	0
$\Phi_2$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	0	$(1, 1, 1)$	0	0	0
$\Phi_3$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	0	$(1, 1, 1)$	0	0	0
$\Phi_{23}$	0	$(1, 1)_0$	-8	0	0	0	48	0	0	24	0	$(1, 1, 1)$	0	0	0
$\bar{\Phi}_{23}$	0	$(1, 1)_0$	8	0	0	0	-48	0	0	-24	0	$(1, 1, 1)$	0	0	0
$\Phi_{13}$	0	$(1, 1)_0$	-28	0	0	4	-32	0	0	84	0	$(1, 1, 1)$	0	0	0
$\bar{\Phi}_{13}$	0	$(1, 1)_0$	28	0	0	-4	32	0	0	-84	0	$(1, 1, 1)$	0	0	0
$\Phi_{12}$	0	$(1, 1)_0$	-36	0	0	4	16	0	0	108	0	$(1, 1, 1)$	0	0	0
$\bar{\Phi}_{12}$	0	$(1, 1)_0$	36	0	0	-4	-16	0	0	-108	0	$(1, 1, 1)$	0	0	0
$\Phi_4$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	4	$(1, 1, 1)$	0	0	0
$\bar{\Phi}_4$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	4	$(1, 1, 1)$	0	0	0
$\bar{\Phi}'_4$	0	$(1, 1)_0$	0	0	0	0	0	0	0	0	-4	$(1, 1, 1)$	0	0	0
$\Phi_{56}$	0	$(1, 1)_0$	8	0	0	0	0	0	8	200	0	$(1, 1, 1)$	0	0	0
$\bar{\Phi}_{56}$	0	$(1, 1)_0$	-8	0	0	0	0	0	-8	-200	0	$(1, 1, 1)$	0	0	0
$\Phi'_{56}$	0	$(1, 1)_0$	0	0	0	0	0	8	0	0	0	$(1, 1, 1)$	0	0	0
$\bar{\Phi}'_{56}$	0	$(1, 1)_0$	0	0	0	0	0	-8	0	0	0	$(1, 1, 1)$	0	0	0

State	$U_E$	$(C, L)_Y$	$U_A$	$U_C$	$U_L$	$U'_1$	$U'_2$	$U'_3$	$U'_4$	$U'_5$	$U_4$	$(3, 2, 2')_H$	$U_7$	$U_H$	$U_9$
$H_{15}^s$	0	$(1, 1)_0$	-8	3	-2	0	-3	-4	-2	136	-2	$(1, 1, 1)$	-1	3	0
$H_{16}^s$	0	$(1, 1)_0$	8	-3	2	0	3	-4	2	-136	2	$(1, 1, 1)$	1	-3	0
$H_{17}^s$	0	$(1, 1)_0$	-4	3	-2	0	-3	0	2	236	2	$(1, 1, 1)$	-1	3	0
$H_{18}^s$	0	$(1, 1)_0$	12	-3	2	0	3	0	6	-36	-2	$(1, 1, 1)$	1	-3	0
$H_{19}^s$	0	$(1, 1)_0$	-16	3	-2	0	-3	4	2	-64	2	$(1, 1, 1)$	-1	3	0
$H_{20}^s$	0	$(1, 1)_0$	16	-3	2	0	3	4	-2	64	-2	$(1, 1, 1)$	1	-3	0
$H_{21}^s$	0	$(1, 1)_0$	-12	3	-2	0	-3	0	6	36	-2	$(1, 1, 1)$	-1	3	0
$H_{22}^s$	0	$(1, 1)_0$	20	-3	2	0	3	0	2	164	2	$(1, 1, 1)$	1	-3	0
$H_{29}^s$	0	$(1, 1)_0$	-4	3	-2	6	-15	-2	0	180	0	$(1, 1, 1)$	-1	3	0
$H_{30}^s$	0	$(1, 1)_0$	-24	-3	2	-2	-17	2	0	-96	0	$(1, 1, 1)$	1	-3	0
$H_{31}^s$	0	$(1, 1)_0$	12	-3	2	6	-9	-2	4	-92	0	$(1, 1, 1)$	1	-3	0
$H_{32}^s$	0	$(1, 1)_0$	0	-3	2	2	11	-2	0	168	0	$(1, 1, 1)$	-3	-3	0
$H_{36}^s$	0	$(1, 1)_0$	20	-3	2	-6	-9	2	0	108	0	$(1, 1, 1)$	1	-3	0
$H_{37}^s$	0	$(1, 1)_0$	16	3	-2	2	-7	-2	0	-216	0	$(1, 1, 1)$	-1	3	0
$H_{38}^s$	0	$(1, 1)_0$	-12	3	-2	-6	-15	2	4	-20	0	$(1, 1, 1)$	-1	3	0
$H_{39}^s$	0	$(1, 1)_0$	8	3	-2	-2	13	2	0	144	0	$(1, 1, 1)$	3	3	0
$V_1^s$	0	$(1, 1)_0$	16	0	0	4	4	0	0	-48	2	$(1, 1, 1)$	2	6	0
$V_2^s$	0	$(1, 1)_0$	16	0	0	4	4	0	0	-48	-2	$(1, 1, 1)$	-2	-6	0
$V_{11}^s$	0	$(1, 1)_0$	16	0	0	-2	16	2	2	8	0	$(1, 1, 1)$	2	6	0
$V_{12}^s$	0	$(1, 1)_0$	12	0	0	-2	16	-2	-2	-92	0	$(1, 1, 1)$	-2	-6	0
$V_{21}^s$	0	$(1, 1)_0$	20	0	0	-2	-8	-2	2	-4	0	$(1, 1, 1)$	2	6	0
$V_{22}^s$	0	$(1, 1)_0$	16	0	0	-2	-8	2	-2	-104	0	$(1, 1, 1)$	-2	-6	0
$V_{31}^s$	0	$(1, 1)_0$	-4	0	0	0	24	0	0	12	2	$(1, 1, 1)$	-2	6	0
$V_{32}^s$	0	$(1, 1)_0$	-4	0	0	0	24	0	0	12	-2	$(1, 1, 1)$	2	-6	0
$\text{tr } Q_s$			168	36	0	0	36	0	24	168	12		0	0	0

Table A.I.b: Same as in Table A.I.a except for Non-Abelian Singlet States. An “s” superscript indicates that the various  $H$  and  $V$  states are singlets.

State	$U_E$	$(C, L)_Y$	$U_A$	$U_C$	$U_L$	$U'_1$	$U'_2$	$U'_3$	$U'_4$	$U'_5$	$U_4$	$(3, 2, 2')_H$	$U_7$	$U_H$	$U_9$
$H_1$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	-4	3	2	2	11	2	2	68	-2	(1,2,1)	1	3	0
$H_2$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	4	-3	-2	-2	-11	-2	-2	-68	-2	(1,2,1)	-1	-3	0
$H_{11}$	$\frac{1}{2}$	$(1, 1)_{\frac{1}{2}}$	0	3	2	2	-13	-2	2	56	2	(1,1,2)	1	3	0
$H_{13}$	$-\frac{1}{2}$	$(1, 1)_{-\frac{1}{2}}$	0	-3	-2	-2	13	2	-2	-56	2	(1,1,2)	-1	-3	0
$H_{42}$	0	$(1, 1)_0$	8	3	-2	-2	13	2	0	144	0	(3,1,1)	-1	-1	0
$V_4$	0	$(1, 1)_0$	16	0	0	4	4	0	0	-48	-2	(3,1,1)	-2	2	0
$V_{14}$	0	$(1, 1)_0$	16	0	0	-2	16	2	2	8	0	(3,1,1)	-2	2	0
$V_{24}$	0	$(1, 1)_0$	20	0	0	-2	-8	-2	2	-4	0	(3,1,1)	-2	2	0
$V_{34}$	0	$(1, 1)_0$	4	0	0	0	-24	0	0	-12	-2	(3,1,1)	2	2	0
$H_{35}$	0	$(1, 1)_0$	0	-3	2	2	11	-2	0	168	0	(3,1,1)	1	1	0
$V_3$	0	$(1, 1)_0$	16	0	0	4	4	0	0	-48	2	(3,1,1)	2	-2	0
$V_{13}$	0	$(1, 1)_0$	12	0	0	-2	16	-2	-2	-92	0	(3,1,1)	2	-2	0
$V_{23}$	0	$(1, 1)_0$	16	0	0	-2	-8	2	-2	-104	0	(3,1,1)	2	-2	0
$V_{33}$	0	$(1, 1)_0$	4	0	0	0	-24	0	0	-12	2	(3,1,1)	-2	-2	0
$H_{25}$	0	$(1, 1)_0$	8	3	-2	-2	-11	-2	2	32	0	(1,2,1)	1	-3	-2
$H_{28}$	0	$(1, 1)_0$	0	-3	2	2	-13	2	2	56	0	(1,2,1)	-1	3	-2
$V_9$	0	$(1, 1)_0$	12	0	0	4	4	0	2	-148	-2	(1,2,1)	0	0	2
$V_{10}$	0	$(1, 1)_0$	20	0	0	4	4	0	-2	52	2	(1,2,1)	0	0	-2
$V_{19}$	0	$(1, 1)_0$	16	0	0	-2	16	-2	-4	8	0	(1,2,1)	0	0	-2
$V_{20}$	0	$(1, 1)_0$	12	0	0	-2	16	2	4	-92	0	(1,2,1)	0	0	2
$V_{29}$	0	$(1, 1)_0$	24	0	0	-2	-8	-2	0	96	0	(1,2,1)	0	0	-2
$V_{30}$	0	$(1, 1)_0$	12	0	0	-2	-8	2	0	-204	0	(1,2,1)	0	0	2
$V_{39}$	0	$(1, 1)_0$	20	0	0	4	4	0	-2	52	2	(1,2,1)	0	0	2
$V_{40}$	0	$(1, 1)_0$	-12	0	0	-4	-4	0	-2	148	2	(1,2,1)	0	0	2
$H_{23}$	0	$(1, 1)_0$	8	3	-2	-2	-11	-2	2	32	0	(1,1,2)	1	-3	2
$H_{26}$	0	$(1, 1)_0$	0	-3	2	2	-13	2	2	56	0	(1,1,2)	-1	3	2
$V_5$	0	$(1, 1)_0$	20	0	0	4	4	0	-2	52	-2	(1,1,2)	0	0	2
$V_7$	0	$(1, 1)_0$	12	0	0	4	4	0	2	-148	2	(1,1,2)	0	0	-2
$V_{15}$	0	$(1, 1)_0$	16	0	0	-2	16	-2	-4	8	0	(1,1,2)	0	0	2
$V_{17}$	0	$(1, 1)_0$	12	0	0	-2	16	2	4	-92	0	(1,1,2)	0	0	-2
$V_{25}$	0	$(1, 1)_0$	24	0	0	-2	-8	-2	0	96	0	(1,1,2)	0	0	2
$V_{27}$	0	$(1, 1)_0$	12	0	0	-2	-8	2	0	-204	0	(1,1,2)	0	0	-2
$V_{35}$	0	$(1, 1)_0$	20	0	0	4	4	0	-2	52	-2	(1,1,2)	0	0	-2
$V_{37}$	0	$(1, 1)_0$	-12	0	0	-4	-4	0	-2	148	-2	(1,1,2)	0	0	-2
$\text{tr } Q_h$			784	0	0	0	0	0	0	0	0		0	0	0

Table A.I.c: Same as in Table A.I.a except for Hidden Sector  $SU(3)_H \times SU(2)_H \times SU(2)'_H$  States (that are  $SU(3)_C \times SU(2)_L$  singlets).

Note: In [8] and [16] both components of several hidden sector  $SU(2)_H$  doublets appeared in the tables of states and in the superpotential terms. The doublets formed by

these pairs of components were  $(H_{11}, H_{12}), (H_{13}, H_{14}), (H_{23}, H_{24}), (H_{26}, H_{27}), (V_5, V_6), (V_7, V_8), (V_{15}, V_{16}), (V_{17}, V_{18}), (V_{25}, V_{26}), (V_{27}, V_{28}), (V_{35}, V_{36}), (V_{37}, V_{38})$ . Throughout this paper, each doublet is denoted by its component with lower subscript.

State	Q						I					
	$x_1$	$y_1$	$\omega_1$	$x_3$	$y_3$	$x_5$	$y_2$	$\omega_2$	$y_4$	$\omega_4$	$y_5$	$\omega_5$
	$x_2$	$\omega_6$	$\omega_3$	$x_4$	$y_6$	$x_6$	$\bar{y}_2$	$\bar{\omega}_2$	$\bar{y}_4$	$\bar{\omega}_4$	$\bar{y}_5$	$\bar{\omega}_5$
$\Phi_1$	-.5			.5		.5	$f$	$f$				
$\Phi_2$	.5			-.5		.5			$\bar{f}$	$\bar{f}$		
$\Phi_3$	.5			.5		-.5					$\bar{f}$	$\bar{f}$
$\Phi_4$	.5			-.5		.5		$f$				
$\bar{\Phi}_4$	.5			-.5		.5		$\bar{f}$				
$\Phi_4'$	.5			.5		-.5	$\bar{f}$					
$\bar{\Phi}_4'$	.5			.5		-.5	$\bar{f}$					
$\Phi_{56}$	-.5			.5		.5						
$\bar{\Phi}_{56}$	-.5			.5		.5						
$\Phi_{56}'$	-.5			.5		.5						
$\bar{\Phi}_{56}'$	-.5			.5		.5						
$\Phi_{23}$	-.5			.5		.5						
$\bar{\Phi}_{23}$	-.5			.5		.5						
$\Phi_{12}$	.5			.5		-.5						
$\bar{\Phi}_{12}$	.5			.5		-.5						
$\Phi_{13}$	.5			-.5		.5						
$\bar{\Phi}_{13}$	.5			-.5		.5						
$H_3^s$		.5		.5				$\sigma^-$			$\sigma^-$	
$H_4^s$		-.5		.5				$\sigma^+$			$\sigma^-$	
$H_5^s$		-.5		.5				$\sigma^-$			$\sigma^-$	
$H_6^s$		.5		.5				$\sigma^+$			$\sigma^-$	
$H_7^s$			.5			.5	$\sigma^+$		$\sigma^+$			
$H_8^s$			-.5			.5	$\sigma^-$		$\sigma^+$			
$H_9^s$			-.5			.5	$\sigma^+$		$\sigma^+$			
$H_{10}^s$			.5			.5	$\sigma^-$		$\sigma^+$			
$H_{15}^s$	.5				-.5				$\sigma^-$			$\sigma^-$
$H_{16}^s$	.5				.5				$\sigma^-$			$\sigma^-$
$H_{17}^s$	.5				-.5				$\sigma^+$			$\sigma^+$
$H_{18}^s$	.5				.5				$\sigma^+$			$\sigma^+$
$H_{19}^s$	.5				-.5					$\sigma^-$	$\sigma^+$	
$H_{20}^s$	.5				.5					$\sigma^-$	$\sigma^+$	
$H_{21}^s$	.5				-.5					$\sigma^+$	$\sigma^-$	
$H_{22}^s$	.5				.5					$\sigma^+$	$\sigma^-$	
$H_{29}^s$		-.5		.5			$\sigma^-$				$\sigma^+$	
$H_{30}^s$		.5		.5			$\sigma^-$				$\sigma^+$	
$H_{31}^s$		.5		.5			$\sigma^+$				$\sigma^+$	
$H_{32}^s$		.5		.5			$\sigma^-$				$\sigma^+$	

State	$Q$						$I$					
	$x_1$	$y_1$	$\omega_1$	$x_3$	$y_3$	$x_5$	$y_2$	$\omega_2$	$y_4$	$\omega_4$	$y_5$	$\omega_5$
	$x_2$	$\omega_6$	$\omega_3$	$x_4$	$y_6$	$x_6$	$\bar{y}_2$	$\bar{\omega}_2$	$\bar{y}_4$	$\bar{\omega}_4$	$\bar{y}_5$	$\bar{\omega}_5$
$H_{36}^s$			.5			.5	$\sigma^+$	$\sigma^-$				
$H_{37}^s$			-.5			.5	$\sigma^+$	$\sigma^-$				
$H_{38}^s$			-.5			.5	$\sigma^-$	$\sigma^-$				
$H_{39}^s$			-.5			.5	$\sigma^+$	$\sigma^-$				
$V_1^s$	.5					.5			$\sigma^+$		$\sigma^+$	
$V_2^s$	.5					-.5			$\sigma^-$		$\sigma^-$	
$V_{11}^s$		.5		.5			$\sigma^+$					$\sigma^+$
$V_{12}^s$		-.5		.5			$\sigma^+$					$\sigma^+$
$V_{21}^s$			.5			.5		$\sigma^-$		$\sigma^+$		
$V_{22}^s$			-.5			.5		$\sigma^-$		$\sigma^+$		
$V_{31}^s$	.5				.5					$\sigma^+$		$\sigma^-$
$V_{32}^s$	.5				-.5					$\sigma^+$		$\sigma^-$
$V_{41}^s$		.5		.5				$\sigma^+$				$\sigma^-$
$V_{42}^s$		-.5		.5				$\sigma^-$				$\sigma^-$
$V_{43}^s$		-.5		.5				$\sigma^+$				$\sigma^-$
$V_{44}^s$		.5		.5				$\sigma^-$				$\sigma^-$
$V_{47}^s$			-.5			.5	$\sigma^+$			$\sigma^-$		
$V_{48}^s$			.5			.5	$\sigma^-$			$\sigma^-$		
$V_{49}^s$			.5			.5	$\sigma^+$			$\sigma^-$		
$V_{50}^s$			-.5			.5	$\sigma^-$			$\sigma^-$		

Table A.II.a: The non-gauge world-sheet charges of the 63 non-Abelian singlets in model FNY, except for the three generations of  $e^c$  and  $N^c$ , whose charges appear in Table A.II.b. The superscript “s” for the  $H$  and  $V$  states denotes non-Abelian singlets. For the heading, in the “ $Q$ ” section the two real fermions specifying a column are the two components of a complex left-moving fermion, and in the “ $I$ ” section they denote the components of a non-chiral Ising fermion. A global  $U(1)$  charge  $Q$  carried by a singlet state is listed in the column of the complex world-sheet fermion associated with the charge. Likewise, a conformal field  $I \in \{f, \bar{f}, \sigma^+, \sigma^-\}$  of a non-chiral Ising fermion carried by a singlet is listed in the column of the appropriate Ising fermion.



State	$Q$						$I$					
	$x_1$	$y_1$	$\omega_1$	$x_3$	$y_3$	$x_5$	$y_2$	$\omega_2$	$y_4$	$\omega_4$	$y_5$	$\omega_5$
	$x_2$	$\omega_6$	$\omega_3$	$x_4$	$y_6$	$x_6$	$\bar{y}_2$	$\bar{\omega}_2$	$\bar{y}_4$	$\bar{\omega}_4$	$\bar{y}_5$	$\bar{\omega}_5$
$Q_1$	.5				-.5				$\sigma^-$		$\sigma^-$	
$d_1^c$	.5				.5				$\sigma^+$		$\sigma^+$	
$u_1^c$	.5				.5				$\sigma^-$		$\sigma^-$	
$L_1$	.5				-.5				$\sigma^+$		$\sigma^+$	
$e_1^c$	.5				.5				$\sigma^-$		$\sigma^-$	
$N_1^c$	.5				.5				$\sigma^+$		$\sigma^+$	
$Q_2$		-.5		.5			$\sigma^+$					$\sigma^+$
$d_2^c$		.5		.5			$\sigma^+$					$\sigma^+$
$u_2^c$		.5		.5			$\sigma^-$					$\sigma^+$
$L_2$		-.5		.5			$\sigma^-$					$\sigma^+$
$e_2^c$		.5		.5			$\sigma^-$					$\sigma^+$
$N_2^c$		.5		.5			$\sigma^+$					$\sigma^+$
$Q_3$			-.5			.5	$\sigma^-$		$\sigma^+$			
$d_3^c$			.5			.5	$\sigma^-$		$\sigma^+$			
$u_3^c$			.5			.5	$\sigma^-$		$\sigma^+$			
$L_3$			.5			.5	$\sigma^-$		$\sigma^+$			
$e_3^c$			.5			.5	$\sigma^+$		$\sigma^+$			
$N_3^c$			.5			.5	$\sigma^+$		$\sigma^+$			
$H_{40}$			-.5			.5	$\sigma^+$	$\sigma^-$				
$H_{33}$		.5		.5			$\sigma^-$				$\sigma^+$	
$h_1$	-.5			.5		.5						
$h_2$	.5			-.5		.5						
$h_3$	.5			.5		-.5						
$\bar{h}_1$	-.5			.5		.5						
$\bar{h}_2$	.5			-.5		.5						
$\bar{h}_3$	.5			.5		-.5						
$H_{34}$		-.5		.5			$\sigma^+$				$\sigma^+$	
$H_{41}$			.5			.5		$\sigma^-$	$\sigma^-$			
$V_{45}$		.5		.5			$\sigma^+$					$\sigma^-$
$V_{46}$		-.5		.5			$\sigma^-$					$\sigma^-$
$V_{51}$			-.5			.5	$\sigma^-$			$\sigma^-$		
$V_{52}$			.5			.5	$\sigma^+$			$\sigma^-$		

Table A.II.b: Same as Table A.II.a except for the non-trivial  $SU(3)_C \times SU(2)_L$  states, along with those for the  $e^c$  and  $N^c$  states.

State	$Q$						$I$					
	$x_1$	$y_1$	$\omega_1$	$x_3$	$y_3$	$x_5$	$y_2$	$\omega_2$	$y_4$	$\omega_4$	$y_5$	$\omega_5$
	$x_2$	$\omega_6$	$\omega_3$	$x_4$	$y_6$	$x_6$	$\bar{y}_2$	$\bar{\omega}_2$	$\bar{y}_4$	$\bar{\omega}_4$	$\bar{y}_5$	$\bar{\omega}_5$
$H_1$		.5		.5				$\sigma^-$			$\sigma^+$	
$H_2$		-.5		.5				$\sigma^+$			$\sigma^+$	
$H_{11}$			.5				$\sigma^+$		$\sigma^-$			
$H_{13}$			-.5			.5	$\sigma^-$		$\sigma^-$			
$H_{23}$		-.5		.5			$\sigma^-$				$\sigma^-$	
$H_{25}$		-.5		.5			$\sigma^-$				$\sigma^-$	
$H_{26}$			-.5			.5		$\sigma^-$	$\sigma^+$			
$H_{28}$			.5			.5		$\sigma^+$	$\sigma^+$			
$H_{35}$		-.5		.5			$\sigma^+$				$\sigma^+$	
$H_{42}$			.5			.5		$\sigma^-$	$\sigma^-$			
$V_3$	.5					.5			$\sigma^-$		$\sigma^-$	
$V_4$	.5				-.5				$\sigma^+$		$\sigma^+$	
$V_5$	.5				.5				$\sigma^-$		$\sigma^+$	
$V_7$	.5				-.5				$\sigma^+$		$\sigma^-$	
$V_9$	.5				.5				$\sigma^+$		$\sigma^-$	
$V_{10}$	.5				-.5				$\sigma^-$		$\sigma^+$	
$V_{13}$		.5		.5			$\sigma^-$					$\sigma^+$
$V_{14}$		-.5		.5			$\sigma^-$					$\sigma^+$
$V_{15}$		.5		.5			$\sigma^+$					$\sigma^-$
$V_{17}$		-.5		.5			$\sigma^+$					$\sigma^-$
$V_{19}$		-.5		.5			$\sigma^-$					$\sigma^-$
$V_{20}$		.5		.5			$\sigma^-$					$\sigma^-$
$V_{23}$			.5			.5		$\sigma^+$		$\sigma^+$		
$V_{24}$			-.5			.5		$\sigma^+$		$\sigma^+$		
$V_{25}$			.5			.5		$\sigma^+$		$\sigma^-$		
$V_{27}$			-.5			.5		$\sigma^+$		$\sigma^-$		
$V_{29}$			-.5			.5		$\sigma^-$		$\sigma^-$		
$V_{30}$			.5			.5		$\sigma^-$		$\sigma^-$		
$V_{33}$	.5					.5				$\sigma^-$		$\sigma^+$
$V_{34}$	.5					.5				$\sigma^+$		$\sigma^-$
$V_{35}$	.5				-.5					$\sigma^+$		$\sigma^+$
$V_{37}$	.5				-.5					$\sigma^-$		$\sigma^-$
$V_{39}$	.5				.5					$\sigma^+$		$\sigma^+$
$V_{40}$	.5				.5					$\sigma^-$		$\sigma^-$

Table A.II.c: Same as Table A.II.a except for the hidden sector non-trivial NA states.

## B Renormalizable and Fourth through Sixth Order Non-renormalizable Terms in FNY Superpotential

Superpotential terms containing only non-Abelian singlet fields (including neutrino  $SU(2)_L$  singlets,  $N_{i=1,2,3}^c$ ). Coupling constants are implicit for non-renormalizable terms.

$W_3(\text{singlets})$ : (with  $g'_s \equiv g_s \sqrt{2}$ )

$$\begin{aligned}
& g_s \left[ \Phi'_4 \{H_7^s H_8^s + H_9^s H_{10}^s\} + \overline{\Phi}_4 \{H_3^s H_4^s + H_5^s H_6^s + V_{41}^s V_{42}^s + V_{43}^s V_{44}^s\} \right. \\
& \quad \left. + \overline{\Phi}_4 \{V_{47}^s V_{48}^s + V_{49}^s V_{50}^s\} \right] \\
& + g'_s \left[ \Phi_1 \{\overline{\Phi}_4 \Phi'_4 + \overline{\Phi}_4' \Phi_4\} + \Phi_{12} \{\overline{\Phi}_{13} \overline{\Phi}_{23} + H_{37}^s H_{36}^s + V_{21}^s V_{22}^s\} + \overline{\Phi}_{12} \Phi_{13} \Phi_{23} \right. \\
& \quad \left. + \Phi_{13} V_{11}^s V_{12}^s + \overline{\Phi}_{13} H_{29}^s H_{30}^s + \overline{\Phi}_{23} V_{31}^s V_{32}^s \right. \\
& \quad \left. + \overline{\Phi}_{56} \{H_{17}^s H_{18}^s + H_{21}^s H_{22}^s\} + \Phi'_{56} H_{15}^s H_{16}^s + \overline{\Phi}'_{56} H_{19}^s H_{20}^s \right]
\end{aligned} \tag{B.1}$$

$W_4(\text{singlets})$ :

$$\begin{aligned}
& N_1^c H_{30}^s H_{32}^s V_1^s + N_3^c H_{30}^s H_{32}^s V_{21}^s + H_3^s H_4^s H_7^s H_8^s + H_3^s H_4^s H_9^s H_{10}^s + H_4^s H_{21}^s V_{32}^s V_{43}^s \\
& + H_5^s H_6^s H_7^s H_8^s + H_5^s H_6^s H_9^s H_{10}^s + H_6^s H_{15}^s V_2^s V_{41}^s + H_7^s H_8^s V_{41}^s V_{42}^s + H_7^s H_8^s V_{43}^s V_{44}^s \\
& + H_9^s H_{10}^s V_{41}^s V_{42}^s + H_9^s H_{10}^s V_{43}^s V_{44}^s + H_{30}^s H_{32}^s H_{37}^s H_{39}^s
\end{aligned} \tag{B.2}$$

$W_5(\text{singlets})$ :

$$\begin{aligned}
& N_1^c \Phi_{12} H_4^s H_{18}^s V_{43}^s + N_2^c \overline{\Phi}_4 H_{22}^s H_{30}^s V_{31}^s + \Phi_1 H_3^s H_4^s H_7^s H_8^s + \Phi_1 H_3^s H_4^s H_9^s H_{10}^s \\
& + \Phi_1 H_5^s H_6^s H_7^s H_8^s + \Phi_1 H_5^s H_6^s H_9^s H_{10}^s + \Phi_1 H_7^s H_8^s V_{41}^s V_{42}^s + \Phi_1 H_7^s H_8^s V_{43}^s V_{44}^s \\
& + \Phi_1 H_9^s H_{10}^s V_{41}^s V_{42}^s + \Phi_1 H_9^s H_{10}^s V_{43}^s V_{44}^s + \overline{\Phi}_{12} H_{29}^s H_{30}^s V_{31}^s V_{32}^s + \Phi_{13} H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + \Phi_{13} V_{21}^s V_{22}^s V_{31}^s V_{32}^s + \Phi_{23} H_4^s H_6^s H_7^s H_9^s + \Phi_{23} H_{29}^s H_{30}^s H_{36}^s H_{37}^s + \Phi_{23} H_{29}^s H_{30}^s V_{21}^s V_{22}^s \\
& + \overline{\Phi}_{23} H_3^s H_5^s H_8^s H_{10}^s + \Phi'_4 H_{15}^s H_{30}^s V_2^s V_{11}^s + \overline{\Phi}'_4 H_{17}^s H_{30}^s V_1^s V_{12}^s + \overline{\Phi}_{56} H_{31}^s H_{32}^s H_{38}^s H_{39}^s
\end{aligned} \tag{B.3}$$

$W_6(\text{singlets})$ :

$$\begin{aligned}
& N_1^c \Phi_3 \Phi_3 H_{30}^s H_{32}^s V_1^s \\
& + N_1^c \Phi_4' \overline{\Phi}_4' H_{30}^s H_{32}^s V_1^s \\
& + N_2^c H_6^s H_7^s H_8^s H_{32}^s V_{41}^s \\
& + N_2^c H_8^s H_{15}^s H_{22}^s H_{30}^s V_{47}^s \\
& + N_3^c \Phi_1 \Phi_1 H_{30}^s H_{32}^s V_{21}^s \\
& + N_3^c \Phi_{56} \overline{\Phi}_{56} H_{30}^s H_{32}^s V_{21}^s \\
& + N_3^c H_4^s H_{15}^s H_{18}^s H_{30}^s V_{47}^s \\
& + \Phi_1 \Phi_1 H_3^s H_4^s H_7^s H_8^s \\
& + \Phi_1 \Phi_1 H_5^s H_6^s H_9^s H_{10}^s \\
& + \Phi_1 \Phi_1 H_9^s H_{10}^s V_{41}^s V_{42}^s \\
& + \Phi_1 \overline{\Phi}_{56} H_{31}^s H_{32}^s H_{38}^s H_{39}^s \\
& + \Phi_3 \Phi_4' H_{15}^s H_{30}^s V_2^s V_{11}^s \\
& + \Phi_{13} \overline{\Phi}_4 H_{16}^s H_{39}^s V_{22}^s V_{31}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_5^s H_6^s H_7^s H_8^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_7^s H_8^s V_{43}^s V_{44}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + \Phi_4' \overline{\Phi}_4' H_6^s H_{15}^s V_2^s V_{41}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_5^s H_6^s H_7^s H_8^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_7^s H_8^s V_{43}^s V_{44}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_5^s H_6^s H_7^s H_8^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_7^s H_8^s V_{43}^s V_{44}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + H_4^s H_6^s H_9^s H_{15}^s V_{32}^s V_{47}^s \\
& + H_5^s H_7^s H_8^s H_{30}^s V_{11}^s V_{42}^s \\
& + H_5^s H_{10}^s H_{21}^s H_{30}^s V_1^s V_{22}^s \\
& + H_6^s H_7^s H_8^s H_{29}^s V_{12}^s V_{41}^s \\
& + H_7^s H_{20}^s H_{30}^s H_{37}^s V_{31}^s V_{44}^s \\
& + H_9^s H_{10}^s H_{17}^s H_{30}^s V_1^s V_{12}^s \\
& + H_{10}^s H_{15}^s H_{19}^s H_{31}^s V_{22}^s V_{41}^s \\
& + H_{15}^s H_{20}^s H_{30}^s H_{37}^s V_{44}^s V_{47}^s \\
& + H_{15}^s H_{30}^s V_2^s V_{11}^s V_{41}^s V_{42}^s \\
& + H_{15}^s H_{30}^s V_2^s V_{11}^s V_{49}^s V_{50}^s \\
& + N_1^c \Phi_3 \Phi_{12} H_4^s H_{18}^s V_{43}^s \\
& + N_1^c H_{16}^s H_{20}^s H_{29}^s H_{30}^s V_{31}^s \\
& + N_2^c H_6^s H_9^s H_{10}^s H_{32}^s V_{41}^s \\
& + N_2^c H_9^s H_{10}^s H_{22}^s H_{30}^s V_{31}^s \\
& + N_3^c \Phi_{13} \overline{\Phi}_4 H_{16}^s H_{36}^s V_{31}^s \\
& + N_3^c \Phi_{56}' \overline{\Phi}_{56}' H_{30}^s H_{32}^s V_{21}^s \\
& + N_3^c H_{10}^s H_{30}^s H_{32}^s H_{37}^s V_{49}^s \\
& + \Phi_1 \Phi_1 H_3^s H_4^s H_9^s H_{10}^s \\
& + \Phi_1 \Phi_1 H_7^s H_8^s V_{41}^s V_{42}^s \\
& + \Phi_1 \Phi_1 H_9^s H_{10}^s V_{43}^s V_{44}^s \\
& + \Phi_3 \Phi_3 H_4^s H_{21}^s V_{32}^s V_{43}^s \\
& + \Phi_{12} \overline{\Phi}_{12} H_4^s H_{21}^s V_{32}^s V_{43}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_3^s H_4^s H_7^s H_8^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_5^s H_6^s H_9^s H_{10}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_9^s H_{10}^s V_{41}^s V_{42}^s \\
& + \Phi_{23} \overline{\Phi}_{56} H_{29}^s H_{31}^s H_{36}^s H_{38}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_3^s H_4^s H_7^s H_8^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_5^s H_6^s H_9^s H_{10}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_9^s H_{10}^s V_{41}^s V_{42}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_3^s H_4^s H_7^s H_8^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_5^s H_6^s H_9^s H_{10}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_9^s H_{10}^s V_{41}^s V_{42}^s \\
& + H_3^s H_4^s H_{15}^s H_{30}^s V_2^s V_{11}^s \\
& + H_4^s H_{15}^s H_{18}^s H_{19}^s V_{32}^s V_{43}^s \\
& + H_5^s H_9^s H_{10}^s H_{30}^s V_{11}^s V_{42}^s \\
& + H_5^s H_{10}^s H_{30}^s H_{38}^s V_1^s V_2^s \\
& + H_6^s H_9^s H_{10}^s H_{29}^s V_{12}^s V_{41}^s \\
& + H_7^s H_{30}^s V_2^s V_{11}^s V_{31}^s V_{48}^s \\
& + H_9^s H_{15}^s H_{22}^s H_{30}^s V_{11}^s V_{50}^s \\
& + H_{10}^s H_{17}^s H_{30}^s H_{37}^s V_2^s V_{41}^s \\
& + H_{15}^s H_{20}^s H_{31}^s H_{38}^s V_{41}^s V_{50}^s \\
& + H_{15}^s H_{30}^s V_2^s V_{11}^s V_{43}^s V_{44}^s \\
& + H_{29}^s H_{30}^s H_{36}^s H_{37}^s V_{31}^s V_{32}^s \\
& + N_1^c \Phi_{12} \overline{\Phi}_{12} H_{30}^s H_{32}^s V_1^s \\
& + N_2^c \Phi_3 \overline{\Phi}_4 H_{22}^s H_{30}^s V_{31}^s \\
& + N_2^c H_7^s H_8^s H_{22}^s H_{30}^s V_{31}^s \\
& + N_2^c H_{10}^s H_{30}^s V_2^s V_{31}^s V_{49}^s \\
& + N_3^c \Phi_{23} \overline{\Phi}_{23} H_{30}^s H_{32}^s V_{21}^s \\
& + N_3^c H_4^s H_7^s H_{18}^s H_{30}^s V_{31}^s \\
& + N_3^c H_{15}^s H_{30}^s H_{31}^s H_{36}^s V_{31}^s \\
& + \Phi_1 \Phi_1 H_5^s H_6^s H_7^s H_8^s \\
& + \Phi_1 \Phi_1 H_7^s H_8^s V_{43}^s V_{44}^s \\
& + \Phi_1 \Phi_1 H_{30}^s H_{32}^s H_{37}^s H_{39}^s \\
& + \Phi_3 \Phi_3 H_6^s H_{15}^s V_2^s V_{41}^s \\
& + \Phi_{12} \overline{\Phi}_{12} H_6^s H_{15}^s V_2^s V_{41}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_3^s H_4^s H_9^s H_{10}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_7^s H_8^s V_{41}^s V_{42}^s \\
& + \Phi_{23} \overline{\Phi}_{23} H_9^s H_{10}^s V_{43}^s V_{44}^s \\
& + \Phi_4' \overline{\Phi}_4' H_4^s H_{21}^s V_{32}^s V_{43}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_3^s H_4^s H_9^s H_{10}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_7^s H_8^s V_{41}^s V_{42}^s \\
& + \Phi_{56} \overline{\Phi}_{56} H_9^s H_{10}^s V_{43}^s V_{44}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_3^s H_4^s H_9^s H_{10}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_7^s H_8^s V_{41}^s V_{42}^s \\
& + \Phi_{56}' \overline{\Phi}_{56}' H_9^s H_{10}^s V_{43}^s V_{44}^s \\
& + H_4^s H_6^s H_7^s H_9^s V_{31}^s V_{32}^s \\
& + H_5^s H_6^s H_{15}^s H_{30}^s V_2^s V_{11}^s \\
& + H_5^s H_{10}^s H_{19}^s H_{30}^s V_2^s V_{21}^s \\
& + H_5^s H_{20}^s H_{21}^s H_{30}^s V_{21}^s V_{50}^s \\
& + H_7^s H_8^s H_{17}^s H_{30}^s V_1^s V_{12}^s \\
& + H_8^s H_{15}^s V_{32}^s V_{42}^s V_{43}^s V_{49}^s \\
& + H_9^s H_{30}^s H_{32}^s H_{39}^s V_{21}^s V_{50}^s \\
& + H_{10}^s H_{21}^s V_2^s V_{41}^s V_{43}^s V_{48}^s \\
& + H_{15}^s H_{30}^s H_{31}^s H_{39}^s V_{22}^s V_{31}^s \\
& + H_{15}^s H_{30}^s V_2^s V_{11}^s V_{47}^s V_{48}^s \\
& + H_{29}^s H_{30}^s V_{21}^s V_{22}^s V_{31}^s V_{32}^s
\end{aligned} \tag{B.4}$$

Superpotential terms containing observable sector  $SU(3)_C \times SU(2)_L$ -charged fields (along with the electron conjugates,  $e_{i=1,2,3}^c$ ). Coupling constants are implicit for non-renormalizable terms.

$W_3(\text{observable})$ :

$$\begin{aligned}
g'_s \quad [ & \bar{h}_1 \{Q_1 u_1^c + N_1^c L_1\} \\
& + h_2 \{Q_2 d_2^c + L_2 e_2^c + H_{34} H_{31}^s + V_{45} V_{43}^s\} + h_3 \{Q_3 d_3^c + L_3 e_3^c + V_{52} V_{47}^s\} \\
& + h_1 \{\bar{h}_2 \Phi_{12} + \bar{h}_3 \Phi_{13}\} + h_2 \{\bar{h}_1 \bar{\Phi}_{12} + \bar{h}_3 \bar{\Phi}_{23}\} + h_3 \{\bar{h}_1 \bar{\Phi}_{13} + \bar{h}_2 \bar{\Phi}_{23}\} \\
& + \bar{h}_2 V_{46} V_{44}^s + \bar{h}_3 \{H_{41} H_{38}^s + V_{51} V_{48}^s\} ] \\
+ g_s \quad [ & V_{45} V_{46} \Phi_4 + V_{51} V_{52} \Phi_4' ]
\end{aligned} \tag{B.5}$$

$W_4(\text{observable})$ :

$$\begin{aligned}
& Q_1 d_3^c H_{41} H_{21}^s + Q_1 L_3 H_{40} H_{21}^s + Q_3 Q_3 H_{33} H_{30}^s + Q_3 u_3^c H_{34} H_{30}^s + Q_3 d_1^c H_{41} H_{19}^s \\
& + u_3^c d_1^c H_{40} H_{19}^s + u_3^c e_3^c H_{33} H_{30}^s + L_1 V_{52} H_7^s H_{19}^s + V_{45} V_{46} V_{47}^s V_{48}^s + V_{45} V_{46} V_{49}^s V_{50}^s \\
& + V_{45} V_{51} V_{41}^s V_{50}^s + V_{46} V_{52} V_{42}^s V_{49}^s + V_{51} V_{52} H_3^s H_4^s + V_{51} V_{52} H_5^s H_6^s + V_{51} V_{52} V_{41}^s V_{42}^s \\
& + V_{51} V_{52} V_{43}^s V_{44}^s
\end{aligned} \tag{B.6}$$

$W_5(\text{observable})$ :

$$\begin{aligned}
& Q_1 Q_2 u_1^c d_2^c \Phi_{12} + Q_1 Q_3 u_1^c d_3^c \Phi_{13} + Q_1 u_1^c L_2 e_2^c \Phi_{12} + Q_1 u_1^c L_3 e_3^c \Phi_{13} \\
& + Q_1 u_1^c \bar{h}_3 H_{29}^s H_{30}^s + Q_1 u_1^c H_{34} \Phi_{12} H_{31}^s + Q_1 u_1^c V_{45} \Phi_{12} V_{43}^s + Q_1 u_1^c V_{52} \Phi_{13} V_{47}^s \\
& + Q_1 u_2^c L_2 e_1^c \Phi_{12} + Q_1 u_2^c \bar{h}_3 H_{15}^s H_{30}^s + Q_1 u_3^c L_3 e_1^c \Phi_{13} + Q_2 u_1^c H_{34} \Phi_{12} H_{16}^s \\
& + Q_2 d_1^c L_1 N_2^c \Phi_{12} + Q_2 d_2^c L_1 N_1^c \Phi_{12} + Q_2 d_2^c h_3 V_{31}^s V_{32}^s + Q_2 d_2^c H_{41} \Phi_{23} H_{38}^s \\
& + Q_2 d_2^c V_{51} \Phi_{23} V_{48}^s + Q_2 L_2 H_{40} \Phi_{23} H_{38}^s + Q_3 u_3^c H_{34} \Phi_1 H_{30}^s + Q_3 u_3^c V_{46} \bar{\Phi}_{23} V_{41}^s \\
& + Q_3 d_1^c L_1 N_3^c \Phi_{13} + Q_3 d_1^c H_{41} \Phi_2 H_{19}^s + Q_3 d_3^c L_1 N_1^c \Phi_{13} + Q_3 d_3^c h_1 H_{29}^s H_{30}^s \\
& + Q_3 d_3^c V_{46} \bar{\Phi}_{23} V_{44}^s + Q_3 L_1 H_{40} \bar{\Phi}_4 H_{19}^s + u_1^c d_3^c H_{40} \Phi_4 H_{21}^s + u_1^c e_2^c H_{33} \Phi_{12} H_{16}^s \\
& + u_2^c d_2^c H_{40} \Phi_{23} H_{38}^s + u_2^c e_1^c H_{33} \Phi_{12} H_{16}^s + u_3^c d_1^c H_{40} \Phi_2 H_{19}^s + d_2^c H_{33} \Phi_4' H_{21}^s V_{32}^s \\
& + L_1 L_2 e_2^c N_1^c \Phi_{12} + L_1 L_3 e_3^c N_1^c \Phi_{13} + L_1 e_3^c H_{41} \bar{\Phi}_4 H_{19}^s + L_1 N_1^c \bar{h}_3 H_{29}^s H_{30}^s \\
& + L_1 N_1^c H_{34} \Phi_{12} H_{31}^s + L_1 N_1^c V_{45} \Phi_{12} V_{43}^s + L_1 N_1^c V_{52} \Phi_{13} V_{47}^s + L_1 N_2^c H_{34} \Phi_{12} H_{18}^s \\
& + L_1 h_2 H_7^s H_9^s V_1^s + L_2 e_2^c h_3 V_{31}^s V_{32}^s + L_2 e_2^c H_{41} \Phi_{23} H_{38}^s + L_2 e_2^c V_{51} \Phi_{23} V_{48}^s \\
& + L_2 N_1^c \bar{h}_3 H_{17}^s H_{30}^s + L_2 N_2^c V_{51} \Phi_{23} V_{49}^s + L_2 \bar{h}_1 H_{38}^s H_{39}^s V_{12}^s + L_2 H_{34} \Phi_4' H_{21}^s V_{32}^s \\
& + L_3 e_1^c H_{41} \Phi_4 H_{21}^s + L_3 e_3^c h_1 H_{29}^s H_{30}^s + L_3 e_3^c V_{46} \bar{\Phi}_{23} V_{44}^s + e_1^c \Phi_{12} H_4^s H_6^s V_2^s \\
& + e_1^c \Phi_{13} H_8^s H_{22}^s V_{50}^s + h_1 V_{52} H_{29}^s H_{30}^s V_{47}^s + h_3 H_{34} H_{31}^s V_{31}^s V_{32}^s + h_3 V_{45} V_{31}^s V_{32}^s V_{43}^s \\
& + h_3 V_{46} H_5^s H_{18}^s V_1^s + \bar{h}_1 H_{41} H_{38}^s V_{11}^s V_{12}^s + \bar{h}_1 V_{46} H_{36}^s H_{37}^s V_{44}^s + \bar{h}_1 V_{46} V_{21}^s V_{22}^s V_{44}^s \\
& + \bar{h}_1 V_{51} V_{11}^s V_{12}^s V_{48}^s + \bar{h}_2 H_{41} H_{38}^s V_{31}^s V_{32}^s + \bar{h}_2 V_{51} V_{31}^s V_{32}^s V_{48}^s + H_{33} H_{40} \Phi_{23} H_{31}^s H_{38}^s \\
& + H_{34} H_{41} \Phi_{23} H_{31}^s H_{38}^s + H_{34} V_{51} \Phi_{23} H_{31}^s V_{48}^s + H_{34} V_{52} \Phi_{56} H_{30}^s V_{50}^s + H_{41} V_{45} \Phi_{23} H_{38}^s V_{43}^s \\
& + H_{41} V_{46} \Phi_{56} H_{37}^s V_{41}^s + V_{45} V_{46} \Phi_1 V_{47}^s V_{48}^s + V_{45} V_{46} \Phi_1 V_{49}^s V_{50}^s + V_{45} V_{51} \Phi_{23} V_{43}^s V_{48}^s \\
& + V_{45} V_{51} \bar{\Phi}_{56} V_{44}^s V_{47}^s + V_{46} V_{52} \bar{\Phi}_{23} V_{44}^s V_{47}^s + V_{46} V_{52} \Phi_{56} V_{43}^s V_{48}^s + V_{51} V_{52} \Phi_1 H_3^s H_4^s \\
& + V_{51} V_{52} \Phi_1 H_5^s H_6^s + V_{51} V_{52} \Phi_1 V_{41}^s V_{42}^s + V_{51} V_{52} \Phi_1 V_{43}^s V_{44}^s
\end{aligned} \tag{B.7}$$

$W_6(\text{observable})$ :

$$\begin{aligned}
& Q_1 Q_1 Q_2 L_2 \Phi_{12} \bar{\Phi}'_4 + Q_1 Q_1 Q_3 L_3 \Phi_{13} \bar{\Phi}_4 + Q_1 Q_1 u_2^c d_2^c \Phi_{12} \bar{\Phi}'_4 \\
& + Q_1 Q_1 u_3^c d_3^c \Phi_{13} \bar{\Phi}_4 + Q_1 Q_1 H_{33} \Phi_{12} \bar{\Phi}'_4 H_{31}^s + Q_1 Q_2 Q_3 L_3 H_{15}^s H_{30}^s \\
& + Q_1 Q_2 u_3^c d_3^c H_{15}^s H_{30}^s + Q_1 Q_2 H_{33} \Phi_{12} \bar{\Phi}'_4 H_{16}^s + Q_1 Q_3 u_1^c d_3^c H_{29}^s H_{30}^s \\
& + Q_1 Q_3 u_2^c d_3^c H_{15}^s H_{30}^s + Q_1 u_1^c L_3 e_3^c H_{29}^s H_{30}^s + Q_1 u_1^c V_{46} H_{29}^s H_{31}^s V_{41}^s \\
& + Q_1 u_1^c V_{52} H_{29}^s H_{30}^s V_{47}^s + Q_1 u_2^c L_3 e_3^c H_{15}^s H_{30}^s + Q_1 u_2^c \bar{h}_3 \Phi_3 H_{15}^s H_{30}^s \\
& + Q_1 u_2^c H_{34} \Phi_{12} \bar{\Phi}'_4 H_{16}^s + Q_1 u_2^c V_{52} H_{15}^s H_{30}^s V_{47}^s + Q_1 u_2^c V_{52} H_7^s H_{30}^s V_{31}^s \\
& + Q_1 u_2^c L_3 e_1^c H_{29}^s H_{30}^s + Q_1 u_3^c L_3 e_2^c H_{15}^s H_{30}^s + Q_1 u_3^c L_3 e_2^c H_{15}^s H_{30}^s \\
& + Q_1 d_1^c L_2 H_{19}^s H_{29}^s V_{32}^s + Q_1 d_1^c H_{41} H_3^s H_8^s H_{29}^s + Q_1 d_2^c L_3 H_{29}^s H_{38}^s V_{32}^s \\
& + Q_1 d_2^c h_3 \Phi_{12} V_2^s V_{11}^s + Q_1 d_3^c L_2 H_{29}^s H_{38}^s V_{32}^s + Q_1 d_3^c L_3 H_7^s H_{21}^s V_{48}^s \\
& + Q_1 d_3^c h_2 \bar{h}_2 H_{41} H_{21}^s + Q_1 d_3^c h_2 \Phi_{13} V_2^s V_{21}^s + Q_1 d_3^c H_{41} \Phi_2 \Phi_2 H_{21}^s \\
& + Q_1 d_3^c H_{41} \Phi_{13} \bar{\Phi}_{13} H_{21}^s + Q_1 d_3^c H_{41} H_{15}^s H_{18}^s H_{19}^s \\
& + Q_1 d_3^c V_{46} H_{18}^s H_{29}^s V_{48}^s + Q_1 L_3 h_2 \bar{h}_2 H_{40} H_{21}^s + Q_1 L_3 H_{40} \Phi_2 \Phi_2 H_{21}^s \\
& + Q_1 L_3 H_{40} \Phi_4 \bar{\Phi}_4 H_{21}^s + Q_1 L_3 H_{40} H_{15}^s H_{18}^s H_{19}^s \\
& + Q_1 H_{40} H_{41} H_8^s H_{21}^s V_{47}^s + Q_1 H_{40} V_{46} H_8^s H_{17}^s H_{31}^s + Q_2 Q_3 Q_3 V_{46} H_4^s H_{30}^s \\
& + Q_2 Q_3 u_1^c H_{40} H_{30}^s V_{31}^s + Q_2 u_1^c L_2 e_1^c \Phi_{12} \bar{\Phi}'_4 + Q_2 u_1^c H_{34} \Phi_3 \Phi_{12} H_{16}^s \\
& + Q_2 u_1^c e_3^c H_{41} H_{30}^s V_{31}^s + Q_2 u_1^c \bar{h}_3 \Phi_4 H_{15}^s H_{30}^s + Q_2 u_2^c H_{34} \Phi_3 \Phi_{12} H_{16}^s \\
& + Q_2 u_1^c V_{46} H_{16}^s H_{29}^s V_{41}^s + Q_2 u_2^c V_{46} H_{15}^s H_{16}^s V_{41}^s + Q_2 u_3^c e_1^c H_{41} H_{30}^s V_{31}^s \\
& + Q_2 u_3^c e_3^c V_{46} H_4^s H_{30}^s + Q_2 d_1^c L_2 N_1^c \Phi_{12} \bar{\Phi}'_4 + Q_2 d_1^c L_2 N_1^c \Phi_{12} \bar{\Phi}'_4 \\
& + Q_2 d_1^c L_3 H_3^s H_{15}^s V_{48}^s + Q_2 d_1^c H_{41} H_{21}^s V_{41}^s V_{50}^s + Q_2 d_2^c L_1 H_{15}^s H_{19}^s V_{32}^s \\
& + Q_2 d_2^c V_{51} V_{31}^s V_{32}^s V_{48}^s + Q_2 d_2^c L_1 H_{15}^s H_{19}^s V_{32}^s + Q_2 d_2^c H_{41} H_{38}^s V_{31}^s V_{32}^s \\
& + Q_2 L_2 H_{40} H_{38}^s V_{31}^s V_{32}^s + Q_2 L_2 H_{40} \Phi_1 \Phi_{23} H_{38}^s \\
& + Q_2 L_2 H_{40} V_{46} H_{18}^s V_{31}^s V_{48}^s + Q_2 H_{40} V_{46} H_8^s H_{16}^s H_{17}^s + Q_2 H_{40} V_{46} H_{18}^s V_{31}^s V_{48}^s \\
& + Q_3 Q_3 h_1 \bar{h}_1 H_{33} H_{30}^s + Q_3 Q_3 H_{33} \Phi_1 \Phi_1 H_{30}^s + Q_3 Q_3 H_{33} \Phi_{23} \bar{\Phi}_{23} H_{30}^s \\
& + Q_3 Q_3 H_{33} \Phi_{56} \bar{\Phi}_{56} H_{30}^s + Q_3 Q_3 H_{33} \Phi_{56} \bar{\Phi}_{56} H_{30}^s + Q_3 u_1^c L_2 H_7^s H_{19}^s V_{41}^s \\
& + Q_3 u_1^c L_3 e_1^c \Phi_{13} \bar{\Phi}_4 + Q_3 u_1^c H_{41} H_{21}^s V_{41}^s V_{43}^s + Q_3 u_2^c e_1^c H_{41} H_{30}^s V_{31}^s \\
& + Q_3 u_1^c V_{51} H_5^s H_{30}^s V_1^s + Q_3 u_1^c V_{52} H_{17}^s H_{30}^s V_{43}^s + Q_3 u_2^c e_2^c H_{41} H_{30}^s V_{31}^s \\
& + Q_3 u_2^c e_3^c V_{46} H_4^s H_{30}^s + Q_3 u_2^c V_{46} H_5^s H_{30}^s V_{21}^s + Q_3 u_3^c e_2^c V_{46} H_4^s H_{30}^s \\
& + Q_3 u_3^c h_1 \bar{h}_1 H_{34} H_{30}^s + Q_3 u_3^c H_{34} \Phi_1 \Phi_1 H_{30}^s + Q_3 u_3^c H_{34} \Phi_{23} \bar{\Phi}_{23} H_{30}^s \\
& + Q_3 u_3^c H_{34} \Phi_{56} \bar{\Phi}_{56} H_{30}^s + Q_3 u_3^c H_{34} \Phi_{56} \bar{\Phi}_{56} H_{30}^s + Q_3 d_1^c L_1 N_3^c H_{29}^s H_{30}^s \\
& + Q_3 d_1^c L_2 N_3^c H_{17}^s H_{30}^s + Q_3 d_1^c L_3 N_2^c H_{17}^s H_{30}^s \\
& + Q_3 d_1^c L_3 H_7^s H_{19}^s V_{48}^s + Q_3 d_1^c h_2 \bar{h}_2 H_{41} H_{19}^s + Q_3 d_1^c h_2 \Phi_{13} V_1^s V_{22}^s \\
& + Q_3 d_1^c H_{41} \Phi_{13} \bar{\Phi}_{13} H_{19}^s + Q_3 d_1^c H_{41} \Phi_4 \bar{\Phi}_4 H_{19}^s + Q_3 d_1^c H_{41} \Phi_4 \bar{\Phi}_4 H_{19}^s \\
& + Q_3 d_2^c L_3 N_1^c H_{17}^s H_{30}^s + Q_3 d_2^c V_{51} H_4^s H_{19}^s V_{32}^s + Q_3 d_3^c L_1 N_1^c H_{29}^s H_{30}^s \\
& + Q_3 d_3^c L_2 N_1^c H_{17}^s H_{30}^s + Q_3 L_1 H_{40} V_{45} V_{46} H_{19}^s + Q_3 L_1 H_{40} V_{51} V_{52} H_{19}^s \\
& + Q_3 L_1 H_{40} \Phi_2 \bar{\Phi}_4 H_{19}^s + Q_3 L_1 H_{40} H_7^s H_8^s H_{19}^s + Q_3 L_1 H_{40} H_9^s H_{10}^s H_{19}^s \\
& + Q_3 H_{40} V_{46} H_4^s H_{18}^s H_{19}^s + Q_3 H_{40} V_{46} H_8^s V_{42}^s V_{49}^s + Q_3 H_{40} V_{51} H_3^s H_4^s H_8^s \\
& + Q_3 H_{40} V_{51} H_5^s H_6^s H_8^s + Q_3 H_{40} V_{51} H_8^s V_{41}^s V_{42}^s + Q_3 H_{40} V_{51} H_8^s V_{43}^s V_{44}^s \\
& + u_1^c u_1^c d_2^c e_2^c \Phi_{12} \bar{\Phi}'_4 + u_1^c u_1^c d_2^c H_3^s H_{29}^s V_{41}^s + u_1^c u_1^c d_2^c H_3^s H_{15}^s V_{41}^s \\
& + u_1^c u_2^c d_2^c e_1^c \Phi_{12} \bar{\Phi}'_4 + u_1^c u_3^c d_2^c H_7^s H_{19}^s V_{41}^s + u_1^c u_3^c d_3^c e_1^c \Phi_{13} \bar{\Phi}_4 \\
& + u_1^c u_3^c d_3^c e_2^c H_{40} H_{30}^s V_{31}^s + u_1^c u_3^c e_2^c H_{40} H_{30}^s V_{31}^s
\end{aligned}$$

$W_6(\text{observable})$  continued:

$$\begin{aligned}
& +u_1^c u_3^c H_{40} H_{21}^s V_{41}^s V_{43}^s \\
& +u_1^c d_2^c d_3^c H_7^s H_{19}^s V_{44}^s \\
& +u_1^c d_3^c H_{40} H_5^s H_6^s H_{21}^s \\
& +u_1^c d_3^c H_{40} H_{21}^s V_{47}^s V_{48}^s \\
& +u_1^c e_2^c H_{33} \Phi_3 \Phi_{12} H_{16}^s \\
& +u_2^c u_3^c e_1^c H_{40} H_{30}^s V_{31}^s \\
& +u_2^c d_1^c d_3^c H_3^s H_{15}^s V_{48}^s \\
& +u_2^c d_2^c H_{40} H_{38}^s V_{31}^s V_{32}^s \\
& +u_3^c d_1^c d_2^c N_3^c H_{17}^s H_{30}^s \\
& +u_3^c d_1^c d_3^c H_7^s H_{19}^s V_{48}^s \\
& +u_3^c d_1^c H_{40} \Phi_{13} \overline{\Phi}_{13} H_{19}^s \\
& +u_3^c e_3^c h_1 \overline{h}_1 H_{33} H_{30}^s \\
& +u_3^c e_3^c H_{33} \Phi_{56} \overline{\Phi}_{56} H_{30}^s \\
& +d_1^c N_2^c H_{33} \Phi_{12} \Phi_4' H_{18}^s \\
& +d_2^c H_{33} V_{51} V_{52} H_3^s V_{44}^s \\
& +d_2^c H_{33} H_3^s H_7^s H_8^s V_{44}^s \\
& +d_2^c H_{33} H_{21}^s V_{32}^s V_{41}^s V_{42}^s \\
& +d_2^c H_{33} H_{21}^s V_{32}^s V_{49}^s V_{50}^s \\
& +d_3^c H_{33} H_{16}^s H_{29}^s H_{38}^s V_{32}^s \\
& +L_1 L_2 e_1^c H_{19}^s H_{29}^s V_{32}^s \\
& +L_1 L_2 H_3^s H_9^s H_{15}^s H_{39}^s \\
& +L_1 e_1^c H_{41} H_3^s H_8^s H_{29}^s \\
& +L_1 e_3^c H_{41} V_{45} V_{46} H_{19}^s \\
& +L_1 e_3^c H_{41} H_7^s H_8^s H_{19}^s \\
& +L_1 N_1^c V_{52} H_{29}^s H_{30}^s V_{47}^s \\
& +L_1 N_3^c V_{45} H_7^s H_{30}^s V_{31}^s \\
& +L_1 N_3^c V_{51} H_{15}^s H_{31}^s V_{41}^s \\
& +L_1 H_{34} H_{41} V_{52} H_3^s H_{21}^s \\
& +L_1 V_{45} H_{15}^s H_{19}^s V_{32}^s V_{43}^s \\
& +L_1 V_{46} H_{29}^s H_{37}^s V_{32}^s V_{49}^s \\
& +L_1 V_{52} \Phi_{13} \overline{\Phi}_{13} H_7^s H_{19}^s \\
& +L_2 L_2 e_1^c H_{17}^s H_{19}^s V_{32}^s \\
& +L_2 L_3 N_1^c H_3^s H_{15}^s V_{49}^s \\
& +L_2 e_2^c V_{51} V_{31}^s V_{32}^s V_{48}^s \\
& +L_2 N_1^c H_{41} H_{21}^s V_{41}^s V_{47}^s \\
& +L_2 N_2^c \overline{h}_1 \Phi_{23} H_{36}^s H_{38}^s \\
& +L_2 N_2^c V_{51} V_{31}^s V_{32}^s V_{49}^s \\
& +L_2 H_{34} \Phi_3 \Phi_4' H_{21}^s V_{32}^s \\
& +L_2 H_{34} H_3^s H_9^s H_{10}^s V_{44}^s \\
& +L_2 H_{34} H_{21}^s V_{32}^s V_{43}^s V_{44}^s \\
& +u_1^c d_1^c d_3^c H_3^s H_{29}^s V_{48}^s \\
& +u_1^c d_2^c H_{40} H_3^s H_8^s H_{17}^s \\
& +u_1^c d_3^c H_{40} H_{21}^s V_{41}^s V_{42}^s \\
& +u_1^c d_3^c H_{40} H_{21}^s V_{49}^s V_{50}^s \\
& +u_1^c H_{33} H_3^s H_{15}^s H_{31}^s V_{41}^s \\
& +u_2^c d_1^c d_2^c N_2^c \Phi_{12} \Phi_4' \\
& +u_2^c d_1^c H_{40} H_{21}^s V_{41}^s V_{50}^s \\
& +u_2^c e_1^c H_{33} \Phi_3 \Phi_{12} H_{16}^s \\
& +u_3^c d_1^c d_3^c N_1^c \Phi_{13} \Phi_4 \\
& +u_3^c d_1^c h_2 \overline{h}_2 H_{40} H_{19}^s \\
& +u_3^c d_1^c H_{40} \Phi_4 \overline{\Phi}_4 H_{19}^s \\
& +u_3^c e_3^c H_{33} \Phi_1 \Phi_1 H_{30}^s \\
& +u_3^c e_3^c H_{33} \Phi_{56}' \overline{\Phi}_{56}' H_{30}^s \\
& +d_1^c H_{33} H_{16}^s H_{19}^s H_{29}^s V_{32}^s \\
& +d_2^c H_{33} \Phi_3 \Phi_4' H_{21}^s V_{32}^s \\
& +d_2^c H_{33} H_3^s H_9^s H_{10}^s V_{44}^s \\
& +d_2^c H_{33} H_{21}^s V_{32}^s V_{43}^s V_{44}^s \\
& +d_3^c N_3^c H_{33} \Phi_{56} \overline{\Phi}_{56}' H_{30}^s \\
& +L_1 L_1 e_2^c N_2^c \Phi_{12} \overline{\Phi}_4 \\
& +L_1 L_2 e_2^c H_{15}^s H_{19}^s V_{32}^s \\
& +L_1 L_3 e_3^c N_1^c H_{29}^s H_{30}^s \\
& +L_1 e_2^c H_{41} H_3^s H_8^s H_{15}^s \\
& +L_1 e_3^c H_{41} V_{51} V_{52} H_{19}^s \\
& +L_1 e_3^c H_{41} H_9^s H_{10}^s H_{19}^s \\
& +L_1 N_2^c \overline{h}_3 \overline{\Phi}_4' H_{17}^s H_{30}^s \\
& +L_1 N_3^c V_{45} H_{15}^s H_{30}^s V_{47}^s \\
& +L_1 h_2 \overline{h}_2 V_{52} H_7^s H_{19}^s \\
& +L_1 H_{34} H_{15}^s H_{19}^s H_{31}^s V_{32}^s \\
& +L_1 V_{46} H_7^s H_9^s V_{31}^s V_{44}^s \\
& +L_1 V_{51} H_{29}^s H_{37}^s V_{32}^s V_{41}^s \\
& +L_1 V_{52} \Phi_4 \overline{\Phi}_4 H_7^s H_{19}^s \\
& +L_2 L_3 e_1^c H_7^s H_{19}^s V_{44}^s \\
& +L_2 e_1^c H_{41} H_3^s H_8^s H_{17}^s \\
& +L_2 e_3^c V_{51} H_4^s H_{19}^s V_{32}^s \\
& +L_2 N_1^c V_{46} H_{17}^s H_{31}^s V_{41}^s \\
& +L_2 N_2^c V_{46} H_{17}^s H_{18}^s V_{41}^s \\
& +L_2 \overline{h}_1 \Phi_1 H_{38}^s H_{39}^s V_{12}^s \\
& +L_2 H_{34} H_3^s H_4^s H_{21}^s V_{32}^s \\
& +L_2 H_{34} H_5^s H_6^s H_{21}^s V_{32}^s \\
& +L_2 H_{34} H_{21}^s V_{32}^s V_{47}^s V_{48}^s \\
& +u_1^c d_2^c d_2^c H_{17}^s H_{19}^s V_{32}^s \\
& +u_1^c d_3^c H_{40} H_3^s H_4^s H_{21}^s \\
& +u_1^c d_3^c H_{40} H_{21}^s V_{43}^s V_{44}^s \\
& +u_1^c e_1^c H_{33} \Phi_{12} \Phi_4' H_{31}^s \\
& +u_1^c H_{40} H_{40} H_{18}^s H_{19}^s V_{31}^s \\
& +u_2^c d_1^c d_2^c N_1^c \Phi_{12} \Phi_4' \\
& +u_2^c d_2^c H_{40} \Phi_1 \Phi_{23} H_{38}^s \\
& +u_3^c d_1^c d_1^c N_3^c \Phi_{13} \Phi_4 \\
& +u_3^c d_1^c d_3^c N_2^c H_{17}^s H_{30}^s \\
& +u_3^c d_1^c H_{40} \Phi_2 \Phi_2 H_{19}^s \\
& +u_3^c d_2^c d_3^c N_1^c H_{17}^s H_{30}^s \\
& +u_3^c e_3^c H_{33} \Phi_{23} \overline{\Phi}_{23} H_{30}^s \\
& +d_1^c N_1^c H_{33} \Phi_{12} \Phi_4' H_{31}^s \\
& +d_2^c N_1^c H_{33} \Phi_{12} \Phi_4' H_{18}^s \\
& +d_2^c H_{33} H_3^s H_4^s H_{21}^s V_{32}^s \\
& +d_2^c H_{33} H_5^s H_6^s H_{21}^s V_{32}^s \\
& +d_2^c H_{33} H_{21}^s V_{32}^s V_{47}^s V_{48}^s \\
& +d_3^c H_{33} H_3^s H_{15}^s H_{18}^s V_{48}^s \\
& +L_1 L_1 e_3^c N_3^c \Phi_{13} \overline{\Phi}_4 \\
& +L_1 L_2 N_3^c H_7^s H_{21}^s V_{41}^s \\
& +L_1 L_3 N_2^c H_7^s H_{21}^s V_{41}^s \\
& +L_1 e_3^c h_2 \Phi_{13} H_{20}^s H_{37}^s \\
& +L_1 e_3^c H_{41} \Phi_2 \overline{\Phi}_4 H_{19}^s \\
& +L_1 N_1^c V_{46} H_{29}^s H_{31}^s V_{41}^s \\
& +L_1 N_2^c V_{46} H_{18}^s H_{29}^s V_{41}^s \\
& +L_1 N_3^c V_{46} H_{15}^s H_{31}^s V_{49}^s \\
& +L_1 H_{33} H_{40} V_{52} H_3^s H_{21}^s \\
& +L_1 H_{41} H_3^s H_9^s H_{15}^s V_{11}^s \\
& +L_1 V_{46} H_9^s H_{16}^s H_{29}^s H_{39}^s \\
& +L_1 V_{52} \Phi_2 \Phi_2 H_7^s H_{19}^s \\
& +L_1 V_{52} H_{29}^s H_{38}^s V_{32}^s V_{43}^s \\
& +L_2 L_3 e_3^c N_1^c H_{17}^s H_{30}^s \\
& +L_2 e_2^c H_{41} H_{38}^s V_{31}^s V_{32}^s \\
& +L_2 N_1^c H_{34} \Phi_{12} \Phi_4' H_{18}^s \\
& +L_2 N_1^c V_{52} H_{17}^s H_{30}^s V_{47}^s \\
& +L_2 N_2^c V_{46} H_{21}^s H_{22}^s V_{41}^s \\
& +L_2 H_{34} V_{51} V_{52} H_3^s V_{44}^s \\
& +L_2 H_{34} H_3^s H_7^s H_8^s V_{44}^s \\
& +L_2 H_{34} H_{21}^s V_{32}^s V_{41}^s V_{42}^s \\
& +L_2 H_{34} H_{21}^s V_{32}^s V_{49}^s V_{50}^s
\end{aligned}$$

$W_6(\text{observable})$  continued:

$$\begin{aligned}
& +L_2 V_{46} H_5^s H_{31}^s H_{38}^s H_{39}^s & +L_2 V_{46} H_9^s H_{16}^s H_{17}^s H_{39}^s & +L_2 V_{46} H_{17}^s H_{37}^s V_{32}^s V_{49}^s \\
& +L_2 V_{51} H_5^s H_{19}^s V_{21}^s V_{32}^s & +L_2 V_{51} H_5^s H_{38}^s V_1^s V_{32}^s & +L_2 V_{51} H_{17}^s H_{37}^s V_{32}^s V_{41}^s \\
& +L_2 V_{52} H_3^s H_{15}^s H_{19}^s H_{36}^s & +L_2 V_{52} H_{17}^s H_{38}^s V_{32}^s V_{43}^s & +L_3 e_1^c h_2 \Phi_{13} H_{22}^s H_{37}^s \\
& +L_3 e_1^c H_{41} H_3^s H_4^s H_{21}^s & +L_3 e_1^c H_{41} H_5^s H_6^s H_{21}^s & +L_3 e_1^c H_{41} H_{21}^s V_{41}^s V_{42}^s \\
& +L_3 e_1^c H_{41} H_{21}^s V_{43}^s V_{44}^s & +L_3 e_1^c H_{41} H_{21}^s V_{47}^s V_{48}^s & +L_3 e_1^c H_{41} H_{21}^s V_{49}^s V_{50}^s \\
& +L_3 e_1^c V_{45} H_{17}^s H_{30}^s V_{50}^s & +L_3 e_1^c V_{46} H_{17}^s H_{31}^s V_{48}^s & +L_3 e_1^c V_{52} H_{17}^s H_{30}^s V_{42}^s \\
& +L_3 N_1^c V_{46} H_{16}^s H_{29}^s V_{49}^s & +L_3 N_1^c V_{51} H_{16}^s H_{29}^s V_{41}^s & +L_3 N_3^c H_{34} \Phi_{56} \overline{\Phi}_{56} H_{30}^s \\
& +L_3 N_3^c V_{51} H_{15}^s H_{16}^s V_{49}^s & +L_3 \bar{h}_2 \Phi_4 H_{15}^s H_{38}^s V_{32}^s & +L_3 H_{34} H_3^s H_{15}^s H_{18}^s V_{48}^s \\
& +L_3 H_{34} H_{16}^s H_{29}^s H_{38}^s V_{32}^s & +L_3 V_{45} H_7^s H_{38}^s V_{41}^s V_{50}^s & +L_3 V_{46} H_7^s H_{37}^s V_{44}^s V_{49}^s \\
& +L_3 V_{51} H_7^s H_{37}^s V_{41}^s V_{44}^s & +L_3 V_{52} H_3^s H_4^s H_7^s H_{38}^s & +L_3 V_{52} H_5^s H_6^s H_7^s H_{38}^s \\
& +L_3 V_{52} H_7^s H_{38}^s V_{41}^s V_{42}^s & +L_3 V_{52} H_7^s H_{38}^s V_{43}^s V_{44}^s & +e_1^c H_{41} H_{41} H_{18}^s H_{19}^s V_{31}^s \\
& +e_1^c H_{41} V_{52} H_6^s H_{19}^s H_{32}^s & +e_1^c \Phi_2 \Phi_{13} H_8^s H_{22}^s V_{50}^s & +e_1^c H_8^s H_{22}^s H_{29}^s H_{30}^s V_{50}^s \\
& +e_2^c H_{41} V_{46} H_8^s H_{16}^s H_{17}^s & +e_2^c H_{41} V_{46} H_{18}^s V_{31}^s V_{48}^s & +e_2^c V_{46} V_{52} H_6^s H_{32}^s V_{48}^s \\
& +e_2^c H_5^s H_8^s H_{10}^s H_{30}^s V_{42}^s & +e_2^c H_8^s H_{15}^s H_{22}^s H_{30}^s V_{50}^s & +e_2^c H_{10}^s H_{30}^s V_2^s V_{31}^s V_{48}^s \\
& +e_3^c H_{41} V_{46} H_4^s H_{18}^s H_{19}^s & +e_3^c H_{41} V_{46} H_8^s V_{42}^s V_{49}^s & +e_3^c H_{41} V_{51} H_3^s H_4^s H_8^s \\
& +e_3^c H_{41} V_{51} H_5^s H_6^s H_8^s & +e_3^c H_{41} V_{51} H_8^s V_{41}^s V_{42}^s & +e_3^c H_{41} V_{51} H_8^s V_{43}^s V_{44}^s \\
& +e_3^c H_4^s H_{10}^s H_{19}^s H_{30}^s V_2^s & +e_3^c H_4^s H_{20}^s H_{21}^s H_{30}^s V_{50}^s & +e_3^c H_8^s H_{29}^s H_{30}^s H_{36}^s V_{50}^s \\
& +N_1^c h_3 \bar{h}_3 H_{30}^s H_{32}^s V_1^s & +N_1^c H_{41} V_{45} H_{30}^s V_{31}^s V_{47}^s & +N_1^c H_{41} V_{46} H_{31}^s V_{31}^s V_{49}^s \\
& +N_1^c H_{41} V_{51} H_{31}^s V_{31}^s V_{41}^s & +N_2^c H_{41} V_{46} H_{18}^s V_{31}^s V_{49}^s & +N_2^c H_{41} V_{51} H_{18}^s V_{31}^s V_{41}^s \\
& +N_2^c V_{45} V_{46} H_{22}^s H_{30}^s V_{31}^s & +N_2^c V_{46} V_{52} H_6^s H_{32}^s V_{49}^s & +N_2^c V_{51} V_{52} H_6^s H_{32}^s V_{41}^s \\
& +N_2^c V_{51} V_{52} H_{22}^s H_{30}^s V_{31}^s & +N_3^c h_1 \bar{h}_1 H_{30}^s H_{32}^s V_{21}^s & +h_1 \bar{h}_1 V_{45} V_{46} V_{47}^s V_{48}^s \\
& +h_1 \bar{h}_1 V_{45} V_{46} V_{49}^s V_{50}^s & +h_1 \bar{h}_1 V_{45} V_{51} V_{41}^s V_{50}^s & +h_1 \bar{h}_1 V_{46} V_{52} V_{42}^s V_{49}^s \\
& +h_1 \bar{h}_1 V_{51} V_{52} H_3^s H_4^s & +h_1 \bar{h}_1 V_{51} V_{52} H_5^s H_6^s & +h_1 \bar{h}_1 V_{51} V_{52} V_{41}^s V_{42}^s \\
& +h_1 \bar{h}_1 V_{51} V_{52} V_{43}^s V_{44}^s & +h_1 \bar{h}_1 H_3^s H_4^s H_7^s H_8^s & +h_1 \bar{h}_1 H_3^s H_4^s H_9^s H_{10}^s \\
& +h_1 \bar{h}_1 H_5^s H_6^s H_7^s H_8^s & +h_1 \bar{h}_1 H_5^s H_6^s H_9^s H_{10}^s & +h_1 \bar{h}_1 H_7^s H_8^s V_{41}^s V_{42}^s \\
& +h_1 \bar{h}_1 H_7^s H_8^s V_{43}^s V_{44}^s & +h_1 \bar{h}_1 H_9^s H_{10}^s V_{41}^s V_{42}^s & +h_1 \bar{h}_1 H_9^s H_{10}^s V_{43}^s V_{44}^s \\
& +h_1 \bar{h}_1 H_{30}^s H_{32}^s H_{37}^s H_{39}^s & +h_3 \bar{h}_3 H_4^s H_{21}^s V_{32}^s V_{43}^s & +h_3 \bar{h}_3 H_6^s H_{15}^s V_2^s V_{41}^s \\
& +h_3 H_{34} \Phi_{12} H_{16}^s V_2^s V_{11}^s & +h_3 H_{34} \Phi_{12} H_{18}^s V_{11}^s V_{12}^s & +h_3 V_{46} \Phi_3 H_5^s H_{18}^s V_1^s \\
& +H_{33} H_{40} \Phi_1 \Phi_{23} H_{31}^s H_{38}^s & +H_{33} H_{40} \Phi_{23} \Phi_{56} H_{30}^s H_{37}^s & +H_{33} H_{40} H_{18}^s H_{21}^s V_{41}^s V_{50}^s \\
& +H_{33} H_{40} H_{31}^s H_{38}^s V_{31}^s V_{32}^s & +H_{34} H_{41} \Phi_{23} \Phi_{56} H_{30}^s H_{37}^s & +H_{34} H_{41} H_{18}^s H_{21}^s V_{41}^s V_{50}^s \\
& +H_{34} H_{41} H_{31}^s H_{38}^s V_{31}^s V_{32}^s & +H_{34} V_{45} H_{30}^s V_{31}^s V_{32}^s V_{42}^s & +H_{34} V_{51} H_{31}^s V_{31}^s V_{32}^s V_{48}^s \\
& +H_{34} V_{52} H_{17}^s H_{18}^s H_{30}^s V_{50}^s & +H_{34} V_{52} H_{21}^s H_{22}^s H_{30}^s V_{50}^s & +H_{41} V_{45} H_{38}^s V_{31}^s V_{32}^s V_{43}^s \\
& +H_{41} V_{46} H_5^s H_{18}^s H_{19}^s V_{21}^s & +H_{41} V_{46} H_5^s H_{18}^s H_{38}^s V_1^s & +H_{41} V_{46} H_5^s H_{31}^s H_{38}^s V_{11}^s \\
& +H_{41} V_{46} H_9^s H_{16}^s H_{17}^s V_{11}^s & +H_{41} V_{46} H_9^s V_{21}^s V_{44}^s V_{47}^s & +H_{41} V_{46} H_{17}^s H_{18}^s H_{37}^s V_{41}^s \\
& +H_{41} V_{46} H_{21}^s H_{22}^s H_{37}^s V_{41}^s & +H_{41} V_{46} H_{21}^s V_1^s V_{12}^s V_{49}^s & +H_{41} V_{51} H_3^s H_5^s H_8^s V_{21}^s \\
& +H_{41} V_{51} H_9^s H_{16}^s H_{39}^s V_{31}^s & +H_{41} V_{51} H_{21}^s V_1^s V_{12}^s V_{41}^s & +H_{41} V_{51} H_{37}^s V_{31}^s V_{32}^s V_{49}^s \\
& +V_{45} V_{46} \Phi_1 \Phi_1 V_{47}^s V_{48}^s & +V_{45} V_{46} \Phi_1 \Phi_1 V_{49}^s V_{50}^s & +V_{45} V_{46} \Phi_{23} \overline{\Phi}_{23} V_{47}^s V_{48}^s \\
& +V_{45} V_{46} \Phi_{23} \overline{\Phi}_{23} V_{49}^s V_{50}^s & +V_{45} V_{46} \Phi_{56} \overline{\Phi}_{56} V_{47}^s V_{48}^s & +V_{45} V_{46} \Phi_{56} \overline{\Phi}_{56} V_{49}^s V_{50}^s \\
& +V_{45} V_{46} \Phi_{56}' \overline{\Phi}_{56}' V_{47}^s V_{48}^s & +V_{45} V_{46} \Phi_{56}' \overline{\Phi}_{56}' V_{49}^s V_{50}^s & +V_{45} V_{46} H_{17}^s H_{30}^s V_1^s V_{12}^s \\
& +V_{45} V_{51} \Phi_1 \Phi_1 V_{41}^s V_{50}^s & +V_{45} V_{51} \Phi_1 \overline{\Phi}_{56} V_{44}^s V_{47}^s & +V_{45} V_{51} \Phi_{23} \overline{\Phi}_{23} V_{41}^s V_{50}^s
\end{aligned}$$



$W_6(\text{observable})$  continued:

$$\begin{aligned}
& +V_{45}V_{51}\Phi_{23}\overline{\Phi}_{56}V_{42}^sV_{49}^s & +V_{45}V_{51}\Phi_{56}\overline{\Phi}_{56}V_{41}^sV_{50}^s & +V_{45}V_{51}\Phi_{56}'\overline{\Phi}_{56}'V_{41}^sV_{50}^s \\
& +V_{45}V_{51}H_5^sH_{30}^sV_{11}^sV_{50}^s & +V_{45}V_{51}V_{31}^sV_{32}^sV_{43}^sV_{48}^s & +V_{46}V_{51}H_5^sH_{18}^sV_1^sV_{48}^s \\
& +V_{46}V_{51}H_5^sH_{31}^sV_{11}^sV_{48}^s & +V_{46}V_{52}\Phi_1\Phi_1V_{42}^sV_{49}^s & +V_{46}V_{52}\Phi_1\Phi_{56}V_{43}^sV_{48}^s \\
& +V_{46}V_{52}\Phi_{23}\overline{\Phi}_{23}V_{42}^sV_{49}^s & +V_{46}V_{52}\overline{\Phi}_{23}\Phi_{56}V_{41}^sV_{50}^s & +V_{46}V_{52}\Phi_{56}\overline{\Phi}_{56}V_{42}^sV_{49}^s \\
& +V_{46}V_{52}\Phi_{56}'\overline{\Phi}_{56}'V_{42}^sV_{49}^s & +V_{46}V_{52}H_6^sH_{29}^sV_{12}^sV_{49}^s & +V_{46}V_{52}H_{16}^sH_{19}^sH_{29}^sH_{36}^s \\
& +V_{46}V_{52}H_{17}^sH_{18}^sV_{43}^sV_{48}^s & +V_{46}V_{52}H_{17}^sH_{22}^sH_{30}^sH_{37}^s & +V_{46}V_{52}H_{21}^sH_{22}^sV_{43}^sV_{48}^s \\
& +V_{51}V_{52}\Phi_1\Phi_1H_3^sH_4^s & +V_{51}V_{52}\Phi_1\Phi_1H_5^sH_6^s & +V_{51}V_{52}\Phi_1\Phi_1V_{41}^sV_{42}^s \\
& +V_{51}V_{52}\Phi_1\Phi_1V_{43}^sV_{44}^s & +V_{51}V_{52}\Phi_{23}\overline{\Phi}_{23}H_3^sH_4^s & +V_{51}V_{52}\Phi_{23}\overline{\Phi}_{23}H_5^sH_6^s \\
& +V_{51}V_{52}\Phi_{23}\overline{\Phi}_{23}V_{41}^sV_{42}^s & +V_{51}V_{52}\Phi_{23}\overline{\Phi}_{23}V_{43}^sV_{44}^s & +V_{51}V_{52}\Phi_{56}\overline{\Phi}_{56}H_3^sH_4^s \\
& +V_{51}V_{52}\Phi_{56}\overline{\Phi}_{56}H_5^sH_6^s & +V_{51}V_{52}\Phi_{56}\overline{\Phi}_{56}V_{41}^sV_{42}^s & +V_{51}V_{52}\Phi_{56}\overline{\Phi}_{56}V_{43}^sV_{44}^s \\
& +V_{51}V_{52}\Phi_{56}'\overline{\Phi}_{56}'H_3^sH_4^s & +V_{51}V_{52}\Phi_{56}'\overline{\Phi}_{56}'H_5^sH_6^s & +V_{51}V_{52}\Phi_{56}'\overline{\Phi}_{56}'V_{41}^sV_{42}^s \\
& +V_{51}V_{52}\Phi_{56}'\overline{\Phi}_{56}'V_{43}^sV_{44}^s & +V_{51}V_{52}H_5^sH_{30}^sV_{11}^sV_{42}^s & +V_{51}V_{52}H_6^sH_{29}^sV_{12}^sV_{41}^s \\
& +V_{51}V_{52}H_{17}^sH_{30}^sV_1^sV_{12}^s
\end{aligned} \tag{B.8}$$

Superpotential terms containing both observable sector  $SU(3)_C \times SU(2)_L$ -charged fields and non-Abelian hidden sector fields. Coupling constants are implicit for non-renormalizable terms.

$W_4(\text{mixed})$ :

$$H_{41}V_{46}H_{11}V_{17} + V_{45}V_{46}H_{11}H_{13} \quad (\text{B.9})$$

$W_5(\text{mixed})$ :

$$\begin{aligned} & Q_1 h_2 H_{40} H_{28} V_9 + Q_3 d_3^c h_2 V_{34} V_{33} + d_1^c H_{33} \Phi_{12} H_{23} V_7 + L_1 h_2 H_9^s H_{11} V_5 \\ & + L_1 h_3 H_5^s H_1 V_9 + L_1 H_{34} \Phi_{12} H_{25} V_9 + L_2 \bar{h}_1 H_{38}^s H_{42} V_{13} + L_2 \bar{h}_3 H_{19}^s H_{23} V_{37} \\ & + L_3 e_3^c h_2 V_{34} V_{33} + e_1^c h_2 H_{41} H_{26} V_7 + e_1^c \Phi_{12} H_6^s H_2 V_{10} + e_1^c \Phi_{13} H_{10}^s H_{13} V_7 \\ & + h_2 V_{52} V_{47}^s V_{34} V_{33} + h_3 V_{46} H_{16}^s H_1 V_{10} + \bar{h}_1 H_{41} H_{38}^s V_{14} V_{13} + \bar{h}_1 H_{41} H_{38}^s V_{19} V_{20} \\ & + \bar{h}_1 H_{41} H_{38}^s V_{15} V_{17} + \bar{h}_1 V_{45} H_{38}^s H_{13} V_{15} + \bar{h}_1 V_{46} V_{44}^s V_{24} V_{23} + \bar{h}_1 V_{46} V_{44}^s V_{29} V_{30} \\ & + \bar{h}_1 V_{46} V_{44}^s V_{25} V_{27} + \bar{h}_1 V_{46} V_{48}^s V_{20} V_{29} + \bar{h}_1 V_{46} V_{48}^s V_{17} V_{25} + \bar{h}_1 V_{51} V_{44}^s V_{19} V_{30} \\ & + \bar{h}_1 V_{51} V_{44}^s V_{15} V_{27} + \bar{h}_1 V_{51} V_{48}^s V_{14} V_{13} + \bar{h}_1 V_{51} V_{48}^s V_{19} V_{20} + \bar{h}_1 V_{51} V_{48}^s V_{15} V_{17} \\ & + \bar{h}_3 V_{45} H_{19}^s H_2 V_{40} + \bar{h}_3 V_{46} V_{44}^s V_{34} V_{33} + V_{45} V_{46} \Phi_1 H_{11} H_{13} \end{aligned} \quad (\text{B.10})$$

$W_6(\text{mixed})$ :

$$\begin{aligned}
& Q_1 u_3^c L_2 H_9^s H_1 V_{40} & + Q_1 u_3^c V_{46} H_{30}^s H_{11} V_{35} & + Q_1 d_2^c h_3 \Phi_{12} V_5 V_{17} \\
& + Q_1 d_3^c h_2 \Phi_{13} V_9 V_{29} & + Q_1 h_2 H_{40} \bar{\Phi}_4 H_{26} V_7 & + Q_1 H_{40} V_{46} V_{44}^s H_{28} V_9 \\
& + Q_1 H_{40} V_{52} H_{21}^s H_{13} H_{26} & + Q_2 d_1^c h_3 \Phi_{12} V_7 V_{15} & + Q_3 u_3^c V_{46} V_{41}^s V_{34} V_{33} \\
& + Q_3 d_1^c h_2 \Phi_{13} V_{10} V_{30} & + Q_3 d_2^c L_3 H_{19}^s H_{23} V_{37} & + Q_3 d_3^c L_2 H_{19}^s H_{23} V_{37} \\
& + Q_3 d_3^c V_{45} H_{19}^s H_2 V_{40} & + Q_3 d_3^c V_{46} V_{44}^s V_{34} V_{33} & + Q_3 L_1 H_{40} H_{19}^s H_1 H_2 \\
& + Q_3 H_{40} V_{46} H_6^s V_7 V_{37} & + Q_3 H_{40} V_{46} V_{44}^s H_{28} V_{30} & + Q_3 H_{40} V_{46} V_{44}^s H_{26} V_{27} \\
& + Q_3 H_{40} V_{46} V_{48}^s H_{28} V_{20} & + Q_3 H_{40} V_{46} V_{48}^s H_{26} V_{17} & + u_1^c d_3^c H_{40} H_{21}^s H_{11} H_{13} \\
& + u_3^c d_2^c d_3^c H_{19}^s H_{23} V_{37} & + u_3^c H_{33} H_9^s H_{16}^s H_1 V_{40} & + d_1^c H_{33} \Phi_{12} \Phi_4' H_{25} V_9 \\
& + d_2^c H_{33} H_{21}^s V_{32}^s H_{11} H_{13} & + L_1 e_2^c h_3 \Phi_{12} V_4 V_{13} & + L_1 e_2^c h_3 \Phi_{12} V_9 V_{19} \\
& + L_1 e_3^c h_2 \Phi_{13} V_4 V_{23} & + L_1 e_3^c h_2 \Phi_{13} V_5 V_{27} & + L_1 e_3^c H_{41} H_{19}^s H_1 H_2 \\
& + L_1 N_2^c V_{52} H_3^s H_{26} V_{37} & + L_1 N_3^c V_{46} H_{31}^s H_{11} V_{37} & + L_1 H_{34} \Phi_{12} \bar{\Phi}_4' H_{23} V_7 \\
& + L_1 H_{41} H_5^s H_{21}^s H_1 V_{30} & + L_1 H_{41} H_5^s H_{38}^s H_1 V_9 & + L_1 V_{45} H_9^s V_{50}^s H_1 V_{40} \\
& + L_1 V_{46} H_5^s H_{21}^s V_{34} V_3 & + L_1 V_{46} H_9^s V_{44}^s H_{11} V_5 & + L_1 V_{46} H_{29}^s V_{32}^s H_{11} V_{27} \\
& + L_1 V_{46} H_{29}^s V_{41}^s H_{25} V_9 & + L_1 V_{51} H_5^s V_{48}^s H_1 V_9 & + L_1 V_{52} H_9^s H_{17}^s V_9 V_{40} \\
& + L_1 V_{52} H_9^s H_{21}^s V_{39} V_{40} & + L_1 V_{52} H_9^s H_{29}^s V_{20} V_{40} & + L_1 V_{52} H_9^s V_{42}^s H_1 V_{40} \\
& + L_2 L_3 e_3^c H_{19}^s H_{23} V_{37} & + L_2 L_3 N_1^c H_3^s H_{11} V_{37} & + L_2 e_1^c h_3 \Phi_{12} V_{14} V_3 \\
& + L_2 e_1^c h_3 \Phi_{12} V_{10} V_{20} & + L_2 N_1^c V_{52} H_3^s H_{28} V_{40} & + L_2 \bar{h}_3 \Phi_3 H_{19}^s H_{23} V_{37} \\
& + L_2 \bar{h}_3 \bar{\Phi}_4' H_{19}^s H_{25} V_{40} & + L_2 H_{34} H_{21}^s V_{32}^s H_{11} H_{13} & + L_2 H_{41} H_3^s H_{38}^s H_1 V_{19} \\
& + L_2 H_{41} H_5^s H_{38}^s H_1 V_{20} & + L_2 V_{45} H_3^s H_{19}^s V_{15} V_{37} & + L_2 V_{45} H_{19}^s V_{12}^s H_1 V_{40} \\
& + L_2 V_{46} H_3^s V_{48}^s H_1 V_{29} & + L_2 V_{46} H_{17}^s V_{32}^s H_{11} V_{27} & + L_2 V_{46} H_{17}^s V_{41}^s H_{25} V_9 \\
& + L_2 V_{46} H_{21}^s V_{41}^s H_{25} V_{39} & + L_2 V_{46} V_{47}^s V_{48}^s H_1 H_{25} & + L_2 V_{46} V_{49}^s V_{50}^s H_1 H_{25} \\
& + L_2 V_{46} H_1 H_{25} H_{11} H_{13} & + L_2 V_{51} H_3^s V_{48}^s H_1 V_{19} & + L_2 V_{51} H_5^s V_{44}^s H_1 V_{30} \\
& + L_2 V_{51} H_5^s V_{48}^s H_1 V_{20} & + L_2 V_{51} V_{41}^s V_{50}^s H_1 H_{25} & + L_2 V_{52} H_3^s H_{19}^s V_{25} V_{37} \\
& + L_2 V_{52} H_9^s H_{17}^s V_{20} V_{40} & + L_2 V_{52} H_{19}^s V_{47}^s H_{23} V_{37} & + L_3 e_1^c h_2 \Phi_{13} V_{24} V_3 \\
& + L_3 e_1^c h_2 \Phi_{13} V_7 V_{25} & + L_3 e_1^c H_{41} H_{21}^s H_{11} H_{13} & + L_3 e_3^c V_{45} H_{19}^s H_2 V_{40} \\
& + L_3 e_3^c V_{46} V_{44}^s V_{34} V_{33} & + L_3 N_3^c V_{51} H_{16}^s H_{11} V_{37} & + L_3 V_{45} H_5^s H_{21}^s V_{30} V_{40} \\
& + L_3 V_{45} H_5^s H_{38}^s V_9 V_{40} & + L_3 V_{46} H_7^s V_{44}^s H_{11} V_{27} & + L_3 V_{46} H_7^s V_{48}^s H_{11} V_{17} \\
& + L_3 V_{46} H_9^s V_{44}^s H_{11} V_{25} & + L_3 V_{46} H_{15}^s V_{49}^s H_{23} V_7 & + L_3 V_{46} H_{11} H_{23} V_7 V_{37} \\
& + L_3 V_{51} H_5^s H_8^s H_{11} H_{23} & + L_3 V_{51} H_9^s V_{44}^s H_{11} V_{15} & + L_3 V_{51} H_{15}^s V_{41}^s H_{23} V_7 \\
& + L_3 V_{52} H_5^s H_{38}^s H_{23} H_{26} & + L_3 V_{52} H_{21}^s V_{43}^s H_{25} V_{40} & + e_1^c h_2 H_{41} \Phi_4 H_{28} V_9 \\
& + e_1^c H_{41} V_{46} V_{44}^s H_{26} V_7 & + e_1^c H_{10}^s H_{29}^s H_{30}^s H_{13} V_7 & + e_1^c H_{30}^s V_{44}^s V_{50}^s H_{26} V_{35} \\
& + e_2^c H_8^s H_{32}^s V_{48}^s V_9 V_{40} & + e_2^c H_{10}^s H_{15}^s H_{30}^s H_{13} V_7 & + e_2^c H_{10}^s H_{30}^s V_{50}^s V_7 V_{37} \\
& + e_3^c H_{41} V_{46} H_6^s V_7 V_{37} & + e_3^c H_{41} V_{46} V_{44}^s H_{28} V_{30} & + e_3^c H_{41} V_{46} V_{44}^s H_{26} V_{27} \\
& + e_3^c H_{41} V_{46} V_{48}^s H_{28} V_{20} & + e_3^c H_{41} V_{46} V_{48}^s H_{26} V_{17} & + e_3^c V_{45} V_{46} V_{48}^s H_{13} H_{26} \\
& + e_3^c H_4^s H_{21}^s H_{30}^s H_{13} V_5 & + e_3^c H_6^s H_{15}^s H_{30}^s H_{13} V_{35} & + e_3^c H_6^s H_{30}^s V_{50}^s V_{35} V_{37} \\
& + e_3^c H_8^s H_{17}^s H_{30}^s H_2 V_{39} & + e_3^c H_8^s H_{29}^s H_{30}^s H_{13} V_{25} & + e_3^c H_{10}^s H_{19}^s H_{30}^s H_2 V_{10} \\
& + e_3^c H_{10}^s H_{29}^s H_{30}^s H_{13} V_{27} & + e_3^c H_{30}^s H_{39}^s V_{50}^s H_2 H_{28} & + e_3^c V_{48}^s H_1 H_2 H_{13} H_{26} \\
& + N_1^c H_3^s H_4^s H_{30}^s V_4 H_{35} & + N_1^c H_5^s H_6^s H_{30}^s V_4 H_{35} & + N_1^c H_5^s H_{10}^s H_{30}^s H_{26} V_7 \\
& + N_1^c H_{15}^s H_{31}^s V_{41}^s H_{13} H_{26} & + N_1^c H_{30}^s H_{32}^s V_{31}^s V_{34} V_3 & + N_1^c H_{30}^s V_{41}^s V_{42}^s V_4 H_{35} \\
& + N_1^c H_{30}^s V_{43}^s V_{44}^s V_4 H_{35} & + N_1^c H_{30}^s V_{47}^s V_{48}^s V_4 H_{35} & + N_1^c H_{30}^s V_{49}^s V_{50}^s V_4 H_{35}
\end{aligned}$$

$W_6(\text{mixed})$  continued:

$$\begin{aligned}
& +N_1^c H_{30}^s V_4 H_{35} H_{11} H_{13} & +N_1^c H_{31}^s V_{41}^s V_{50}^s H_{26} V_{37} & +N_2^c H_3^s H_{10}^s H_{30}^s H_{28} V_{19} \\
& +N_2^c H_3^s H_{10}^s V_{41}^s H_2 H_{28} & +N_2^c H_5^s H_{10}^s H_{30}^s H_{28} V_{20} & +N_2^c H_5^s H_{10}^s H_{30}^s H_{26} V_{17} \\
& +N_2^c H_8^s H_{32}^s V_{41}^s H_{23} H_{26} & +N_2^c H_8^s H_{32}^s V_{49}^s V_9 V_{40} & +N_2^c H_{10}^s H_{30}^s V_{47}^s V_7 V_{37} \\
& +N_2^c H_{15}^s H_{18}^s V_{41}^s H_{13} H_{26} & +N_2^c H_{18}^s V_{41}^s V_{50}^s H_{26} V_{37} & +N_2^c H_{22}^s H_{30}^s V_{31}^s H_1 H_2 \\
& +N_3^c h_1 \bar{h}_1 H_{30}^s V_{24} H_{35} & +N_3^c h_2 \bar{h}_2 H_{16}^s H_{26} V_{37} & +N_3^c V_{45} V_{46} H_{16}^s H_{28} V_{40} \\
& +N_3^c V_{51} V_{52} H_{16}^s H_{28} V_{40} & +N_3^c \Phi_1 \Phi_1 H_{30}^s V_{24} H_{35} & +N_3^c \Phi_2 \Phi_2 H_{16}^s H_{26} V_{37} \\
& +N_3^c \Phi_2 \Phi_4 H_{16}^s H_{28} V_{40} & +N_3^c \Phi_{13} \Phi_{13} H_{16}^s H_{26} V_{37} & +N_3^c \Phi_{23} \Phi_{23} H_{30}^s V_{24} H_{35} \\
& +N_3^c \Phi_4 \Phi_4 H_{16}^s H_{26} V_{37} & +N_3^c \Phi_{56} \Phi_{56} H_{30}^s V_{24} H_{35} & +N_3^c \Phi_{56}' \Phi_{56}' H_{30}^s V_{24} H_{35} \\
& +N_3^c H_7^s H_8^s H_{16}^s H_{28} V_{40} & +N_3^c H_8^s H_{30}^s H_{32}^s H_{11} V_{25} & +N_3^c H_9^s H_{10}^s H_{16}^s H_{28} V_{40} \\
& +N_3^c H_{10}^s H_{30}^s H_{32}^s H_{11} V_{27} & +N_3^c H_{16}^s H_1 H_2 H_{28} V_{40} & +N_3^c H_{30}^s H_{31}^s V_{31}^s V_{25} V_{37} \\
& +h_1 \bar{h}_1 H_{41} V_{46} H_{11} V_{17} & +h_1 \bar{h}_1 V_{45} V_{46} H_{11} H_{13} & +h_1 \bar{h}_1 H_5^s H_8^s H_{23} H_{26} \\
& +h_1 \bar{h}_1 H_9^s V_{44}^s H_{26} V_{15} & +h_1 \bar{h}_1 H_{30}^s H_{37}^s H_{42} H_{35} & +h_1 \bar{h}_1 H_{30}^s V_{50}^s H_1 V_{29} \\
& +h_1 \bar{h}_1 H_{32}^s V_{48}^s H_1 V_{30} & +h_1 \bar{h}_1 V_{47}^s V_{48}^s H_1 H_2 & +h_1 \bar{h}_1 V_{49}^s V_{50}^s H_1 H_2 \\
& +h_1 \bar{h}_1 H_1 H_2 H_{11} H_{13} & +h_2 \bar{h}_2 H_{37}^s V_{32}^s H_{28} V_{40} & +h_3 \bar{h}_3 H_{15}^s H_{30}^s V_{14} V_3 \\
& +h_3 \bar{h}_3 H_{15}^s H_{30}^s V_{10} V_{20} & +h_3 \bar{h}_3 H_{17}^s H_{30}^s V_4 V_{13} & +h_3 \bar{h}_3 H_{17}^s H_{30}^s V_9 V_{19} \\
& +h_3 \bar{h}_3 H_{17}^s V_{41}^s H_2 V_9 & +h_3 \bar{h}_3 H_{19}^s H_{31}^s V_{15} V_{37} & +h_3 \bar{h}_3 H_{19}^s H_{32}^s V_{34} V_{13} \\
& +h_3 \bar{h}_3 H_{19}^s V_{12}^s V_{34} H_{35} & +h_3 \bar{h}_3 H_{21}^s H_{30}^s V_{19} V_{39} & +h_3 \bar{h}_3 H_{21}^s V_{41}^s H_2 V_{39} \\
& +h_3 \bar{h}_3 H_{30}^s V_{31}^s H_{23} V_{35} & +h_3 H_{34} \Phi_{12} H_{16}^s V_5 V_{17} & +h_3 H_{34} \Phi_{12} H_{18}^s V_7 V_{15} \\
& +\bar{h}_1 V_{45} \Phi_1 H_{38}^s H_{13} V_{15} & +\bar{h}_1 V_{46} \Phi_1 V_{48}^s V_{17} V_{25} & +\bar{h}_1 V_{46} \Phi_{56} V_{48}^s V_{19} V_{30} \\
& +\bar{h}_1 V_{46} \Phi_{56} V_{48}^s V_{15} V_{27} & +\bar{h}_1 V_{51} \Phi_1 V_{44}^s V_{15} V_{27} & +\bar{h}_1 V_{51} \Phi_{23} H_4^s H_{25} V_{30} \\
& +\bar{h}_1 V_{51} \Phi_{23} H_4^s H_{23} V_{27} & +\bar{h}_1 V_{51} \Phi_{56} V_{44}^s V_{20} V_{29} & +\bar{h}_1 V_{51} \Phi_{56} V_{44}^s V_{17} V_{25} \\
& +H_{41} V_{45} H_9^s V_{31}^s V_{20} V_{40} & +H_{41} V_{46} \Phi_1 \Phi_1 H_{11} V_{17} & +H_{41} V_{46} \Phi_{23} \Phi_{23} H_{11} V_{17} \\
& +H_{41} V_{46} \Phi_{56} \Phi_{56} H_{11} V_{17} & +H_{41} V_{46} \Phi_{56}' \Phi_{56}' H_{11} V_{17} & +H_{41} V_{46} H_3^s V_{21}^s V_7 V_{37} \\
& +H_{41} V_{46} H_{16}^s H_{19}^s H_1 V_{29} & +H_{41} V_{46} H_{16}^s H_{38}^s H_1 V_{10} & +H_{41} V_{46} H_{21}^s V_{41}^s V_{24} V_3 \\
& +H_{41} V_{46} H_{21}^s V_{41}^s V_7 V_{25} & +H_{41} V_{46} H_{21}^s V_{49}^s V_7 V_{15} & +H_{41} V_{46} V_{31}^s V_{49}^s H_{25} V_9 \\
& +H_{41} V_{51} H_{16}^s H_{19}^s H_1 V_{19} & +H_{41} V_{51} H_{21}^s V_{41}^s V_7 V_{15} & +H_{41} V_{51} V_{31}^s V_{32}^s H_{11} V_{27} \\
& +H_{41} V_{51} V_{31}^s V_{41}^s H_{25} V_9 & +V_{45} V_{46} \Phi_1 \Phi_1 H_{11} H_{13} & +V_{45} V_{46} \Phi_{23} \Phi_{23} H_{11} H_{13} \\
& +V_{45} V_{46} \Phi_{56} \Phi_{56} H_{11} H_{13} & +V_{45} V_{46} \Phi_{56}' \Phi_{56}' H_{11} H_{13} & +V_{45} V_{46} H_{17}^s H_{30}^s V_7 V_{15} \\
& +V_{45} V_{46} H_{19}^s H_{31}^s V_{19} V_{40} & +V_{45} V_{46} H_{19}^s H_{35} V_{13} V_{33} & +V_{45} V_{46} H_{30}^s V_{31}^s H_{25} V_{39} \\
& +V_{45} V_{52} H_{19}^s H_{32}^s V_{30} V_{40} & +V_{45} V_{52} H_{19}^s V_{47}^s H_2 V_{40} & +V_{46} V_{51} H_4^s V_{32}^s H_{11} V_7 \\
& +V_{46} V_{51} H_{16}^s V_{48}^s H_1 V_{10} & +V_{46} V_{52} H_{17}^s H_{30}^s V_{24} V_3 & +V_{46} V_{52} H_{17}^s H_{30}^s V_7 V_{25} \\
& +V_{46} V_{52} H_{19}^s H_{31}^s V_{29} V_{40} & +V_{46} V_{52} H_{31}^s H_{38}^s V_{10} V_{40} & +V_{46} V_{52} H_{38}^s H_{35}^s V_3 V_{33} \\
& +V_{46} V_{52} V_{22}^s V_{44}^s H_{11} H_{26} & +V_{46} V_{52} V_{44}^s V_{47}^s V_{34} V_{33} & +V_{46} V_{52} H_{25} V_{40} H_{26} V_7 \\
& +V_{46} V_{52} H_{28} V_{40} H_{23} V_7 & +V_{51} V_{52} H_{17}^s H_{30}^s V_7 V_{15} & +V_{51} V_{52} H_{19}^s H_{31}^s V_{19} V_{40} \\
& +V_{51} V_{52} H_{19}^s H_{35} V_{13} V_{33} & +V_{51} V_{52} H_{30}^s V_{31}^s H_{25} V_{39} & 
\end{aligned} \tag{B.11}$$

Superpotential terms containing non-Abelian hidden sector fields. Coupling constants are implicit for non-renormalizable terms.

$W_3(\text{hidden})$ :

$$g'_s \quad [ \Phi_{12} \{ V_{23} V_{24} + V_{29} V_{30} + V_{25} V_{27} \} + \Phi_{13} \{ V_{13} V_{14} + V_{19} V_{20} + V_{15} V_{17} \} + \Phi_{23} V_{33} V_{34} ] \\ + g_s \quad [ \Phi_4 H_1 H_2 + \overline{\Phi}_4' H_{11} H_{13} ] \quad (\text{B.12})$$

$W_4(\text{hidden})$ :

$$\begin{aligned} & N_3^c V_{24} H_{35} H_{30}^s + N_3^c H_{26} V_{37} H_{16}^s + H_{42} H_{35} H_{30}^s H_{37}^s + V_4 V_{13} H_{17}^s H_{30}^s + V_{14} V_3 H_{15}^s H_{30}^s \\ & + V_{34} H_{35} H_{19}^s V_{12}^s + V_{34} V_{13} H_{19}^s H_{32}^s + H_1 H_2 H_{11} H_{13} + H_1 H_2 V_{47}^s V_{48}^s + H_1 H_2 V_{49}^s V_{50}^s \\ & + H_1 V_{29} H_{30}^s V_{50}^s + H_1 V_{30} H_{32}^s V_{48}^s + H_2 V_9 H_{17}^s V_{41}^s + H_2 V_{39} H_{21}^s V_{41}^s + H_{28} V_{40} H_{37}^s V_{32}^s \\ & + V_9 V_{19} H_{17}^s H_{30}^s + V_{10} V_{20} H_{15}^s H_{30}^s + V_{19} V_{39} H_{21}^s H_{30}^s + H_{23} H_{26} H_5^s H_8^s + H_{23} V_{35} H_{30}^s V_{31}^s \\ & + H_{26} V_{15} H_9^s V_{44}^s + V_{15} V_{37} H_{19}^s H_{31}^s \end{aligned} \quad (\text{B.13})$$

$W_5(\text{hidden})$ :

$$\begin{aligned} & N_1^c V_4 H_{35} \Phi_4' H_{30}^s + N_3^c V_{24} H_{35} \Phi_1 H_{30}^s + N_3^c H_{28} V_{40} \overline{\Phi}_4 H_{16}^s + N_3^c H_{26} V_{37} \Phi_2 H_{16}^s \\ & + H_{42} H_{35} \Phi_1 H_{30}^s H_{37}^s + H_{42} H_{35} \overline{\Phi}_{56} H_{31}^s H_{38}^s + V_{14} V_{34} V_{13} V_{33} \Phi_{12} + V_{14} V_3 \Phi_3 H_{15}^s H_{30}^s \\ & + V_{24} V_{23} \Phi_{13} V_{31}^s V_{32}^s + V_{24} V_{23} \Phi_{23} H_{29}^s H_{30}^s + V_{34} V_{33} V_{19} V_{20} \Phi_{12} + V_{34} V_{33} V_{15} V_{17} \Phi_{12} \\ & + V_{34} V_{33} \Phi_{12} V_{11}^s V_{12}^s + H_{35} V_{13} V_{33} \overline{\Phi}_4' H_{19}^s + H_1 H_2 H_{11} H_{13} \Phi_1 + H_1 H_2 \Phi_1 V_{47}^s V_{48}^s \\ & + H_1 H_2 \Phi_1 V_{49}^s V_{50}^s + H_2 V_9 \Phi_3 H_{17}^s V_{41}^s + H_2 V_{30} \Phi_{23} H_{29}^s V_{49}^s + H_2 V_{39} \Phi_3 H_{21}^s V_{41}^s \\ & + H_{25} H_{28} \overline{\Phi}_{56} H_3^s H_{10}^s + H_{25} V_{39} \overline{\Phi}_4' H_{30}^s V_{31}^s + H_{28} V_{20} \overline{\Phi}_{56} H_7^s V_{44}^s + H_{28} V_{30} \Phi_4 H_{15}^s V_{32}^s \\ & + H_{28} V_{40} \Phi_2 H_{37}^s V_{32}^s + V_9 V_{19} \Phi_3 H_{17}^s H_{30}^s + V_{19} V_{39} \Phi_3 H_{21}^s H_{30}^s + V_{19} V_{40} \overline{\Phi}_4' H_{19}^s H_{31}^s \\ & + V_{29} V_{30} \Phi_{13} V_{31}^s V_{32}^s + V_{29} V_{30} \Phi_{23} H_{29}^s H_{30}^s + H_{23} H_{26} \Phi_{23} H_4^s H_9^s + H_{23} V_{35} \Phi_3 H_{30}^s V_{31}^s \\ & + H_{26} V_{17} \overline{\Phi}_{56} H_7^s V_{44}^s + H_{26} V_{27} \Phi_4 H_{15}^s V_{32}^s + H_{26} V_{37} \Phi_4 H_{37}^s V_{32}^s + V_5 V_{17} \Phi_4' H_{15}^s H_{30}^s \\ & + V_7 V_{15} \overline{\Phi}_4' H_{17}^s H_{30}^s + V_{15} V_{35} \Phi_4' H_{21}^s H_{30}^s + V_{25} V_{27} \Phi_{13} V_{31}^s V_{32}^s + V_{25} V_{27} \Phi_{23} H_{29}^s H_{30}^s \end{aligned} \quad (\text{B.14})$$

$W_6(\text{hidden})$ :

$$\begin{aligned}
& H_{42}H_{35}\Phi_1\Phi_1H_{30}^sH_{37}^s + H_{42}H_{35}\Phi_{23}\overline{\Phi}_{23}H_{30}^sH_{37}^s + H_{42}H_{35}\Phi_{56}\overline{\Phi}_{56}H_{30}^sH_{37}^s \\
& + H_{42}H_{35}\Phi_{56}'\overline{\Phi}_{56}'H_{30}^sH_{37}^s + H_{42}H_{35}H_9^sH_{30}^sV_{21}^sV_{50}^s + H_{42}V_{13}H_{26}V_7H_{15}^sH_{30}^s \\
& + H_{42}V_{23}\Phi_{13}\overline{\Phi}_4H_{16}^sV_{31}^s + H_{42}V_{23}H_{15}^sH_{30}^sH_{31}^sV_{31}^s + V_4H_{35}V_{20}V_{40}H_9^sV_{48}^s \\
& + V_4H_{35}V_{30}V_{40}H_9^sV_{44}^s + V_4H_{35}H_{38}^sV_{32}^sV_{43}^sV_{48}^s + V_4V_3H_2V_{40}H_{21}^sV_{41}^s \\
& + V_4V_3V_{19}V_{40}H_{21}^sH_{30}^s + V_4V_3H_{23}V_{37}H_{30}^sV_{31}^s + V_4V_3H_5^sH_{10}^sH_{30}^sH_{38}^s \\
& + V_4V_{13}V_{10}V_{40}H_{21}^sH_{30}^s + V_4V_{13}H_{26}V_{37}H_3^sV_{48}^s + V_4V_{13}\Phi_3\Phi_3H_{17}^sH_{30}^s \\
& + V_4V_{13}\Phi_{12}\overline{\Phi}_{12}H_{17}^sH_{30}^s + V_4V_{13}\Phi_4'\overline{\Phi}_4'H_{17}^sH_{30}^s + V_4V_{13}H_{15}^sH_{19}^sH_{22}^sH_{30}^s \\
& + V_4V_{23}H_7^sH_{30}^sV_{31}^sV_{44}^s + V_4V_{23}H_{15}^sH_{30}^sV_{44}^sV_{47}^s + V_4V_{33}H_6^sH_{15}^sV_{32}^sV_{41}^s \\
& + V_{14}V_{34}V_{34}\Phi_{12}\Phi_{12}'V_{12}^s + V_{14}H_{35}V_9V_{40}H_9^sV_{48}^s + V_{14}V_3\Phi_3\Phi_3H_{15}^sH_{30}^s \\
& + V_{14}V_3\Phi_{12}\overline{\Phi}_{12}H_{15}^sH_{30}^s + V_{14}V_3\Phi_4'\overline{\Phi}_4'H_{15}^sH_{30}^s + V_{14}V_{33}H_{15}^sH_{30}^sV_1^sV_{32}^s \\
& + V_{24}H_{35}H_9^sH_{30}^sH_{39}^sV_{50}^s + V_{24}V_{23}H_{29}^sH_{30}^sV_{31}^sV_{32}^s + V_{34}H_{35}H_2V_{19}H_3^sH_{19}^s \\
& + V_{34}H_{35}H_2V_{20}H_5^sH_{19}^s + V_{34}H_{35}H_{13}H_{23}H_9^sV_{31}^s + V_{34}H_{35}H_{13}V_{27}H_3^sH_{15}^s \\
& + V_{34}H_{35}H_{13}V_{37}H_3^sH_{37}^s + V_{34}H_{35}V_{27}V_{37}H_3^sV_{50}^s + V_{34}H_{35}\Phi_3\Phi_3H_{19}^sV_{12}^s \\
& + V_{34}H_{35}\Phi_{12}\overline{\Phi}_{12}H_{19}^sV_{12}^s + V_{34}H_{35}\Phi_4'\overline{\Phi}_4'H_{19}^sV_{12}^s + V_{34}H_{35}H_{21}^sV_{22}^sV_{41}^sV_{50}^s \\
& + V_{34}H_{35}H_{38}^sV_2^sV_{41}^sV_{50}^s + V_{34}V_{13}\Phi_{12}\overline{\Phi}_{12}H_{19}^sH_{32}^s + V_{34}V_{13}\Phi_3\Phi_3H_{19}^sH_{32}^s \\
& + V_{34}V_{23}H_{21}^sH_{32}^sV_{41}^sV_{50}^s + V_{34}V_{33}H_3^sH_5^sH_8^sH_{10}^s + V_{34}V_{13}H_{17}^sH_{30}^sV_2^sV_{31}^s \\
& + H_{35}V_{13}V_{33}H_1^sH_2^sH_{19}^s + H_{35}V_{13}V_{33}H_7^sH_8^sH_{19}^s + H_{35}H_{35}V_{23}H_9^sH_{38}^sV_{50}^s \\
& + H_{35}V_{23}V_{33}H_3^sH_8^sH_{15}^s + V_{13}V_{33}V_{33}\Phi_{12}\overline{\Phi}_{12}'V_{11}^s + H_{35}V_{13}V_{33}H_9^sH_{10}^sH_{19}^s \\
& + H_1H_2H_{25}V_{39}H_{30}^sV_{31}^s + H_1H_2V_{19}V_{40}H_{19}^sH_{31}^s + H_1H_2H_{11}H_{13}\Phi_{23}\overline{\Phi}_{23} \\
& + H_1H_2H_{11}H_{13}\Phi_{56}\overline{\Phi}_{56} + H_1H_2H_{11}H_{13}\Phi_{56}'\overline{\Phi}_{56}' \\
& + H_1H_2V_7V_{15}H_{17}^sH_{30}^s + H_1H_2\Phi_1\Phi_1V_{47}^sV_{48}^s + H_1H_2\Phi_1\Phi_1V_{49}^sV_{50}^s \\
& + H_1H_2\Phi_{23}\overline{\Phi}_{23}V_{47}^sV_{48}^s + H_1H_2\Phi_{23}\overline{\Phi}_{23}V_{49}^sV_{50}^s + H_1H_2\Phi_{56}\overline{\Phi}_{56}V_{47}^sV_{48}^s \\
& + H_1H_2\Phi_{56}'\overline{\Phi}_{56}'V_{47}^sV_{48}^s + H_1H_2\Phi_{56}\overline{\Phi}_{56}V_{49}^sV_{50}^s + H_1H_2\Phi_{56}'\overline{\Phi}_{56}'V_{49}^sV_{50}^s \\
& + H_1H_2H_{17}^sH_{30}^sV_{12}^sV_{12}^s + H_1H_{28}V_9V_{40}V_{12}^sV_{48}^s + H_1V_9H_4^sH_{17}^sH_{30}^sV_{12}^s \\
& + H_1V_{10}H_6^sH_{15}^sH_{30}^sV_{12}^s + H_1V_{29}\Phi_1\Phi_1H_{30}^sV_{50}^s + H_1V_{29}\Phi_{23}\overline{\Phi}_{23}H_{30}^sV_{50}^s \\
& + H_1V_{29}\Phi_{56}\overline{\Phi}_{56}H_{30}^sV_{50}^s + H_1V_{30}\Phi_{23}\overline{\Phi}_{23}H_{32}^sV_{48}^s + H_1V_{30}\Phi_{56}\overline{\Phi}_{56}H_{32}^sV_{48}^s \\
& + H_1V_{30}\Phi_{56}'\overline{\Phi}_{56}'H_{32}^sV_{48}^s + H_1V_{39}H_4^sH_{21}^sH_{30}^sV_{12}^s + H_1V_{40}H_{13}V_{27}H_{15}^sH_{31}^s \\
& + H_1V_{40}H_{13}V_{37}H_{31}^sH_{37}^s + H_1V_{40}V_{27}V_{37}H_{31}^sV_{50}^s + H_1V_{40}H_4^sH_9^sH_{18}^sV_{50}^s \\
& + H_1V_{40}H_8^sH_{16}^sH_{29}^sV_{22}^s + H_2H_2H_{11}H_{11}\Phi_{23}\Phi_{56}' + H_2H_{28}H_{11}V_{15}^sH_{19}^sV_{32}^s \\
& + H_2V_9V_{10}V_{40}H_{21}^sV_{41}^s + H_2V_9\Phi_3\Phi_3H_{17}^sV_{41}^s + H_2V_9\Phi_{12}\overline{\Phi}_{12}H_{17}^sV_{41}^s \\
& + H_2V_9\Phi_4'\overline{\Phi}_4'H_{17}^sV_{41}^s + H_2V_9H_5^sH_{10}^sH_{19}^sV_{49}^s + H_2V_9H_5^sH_{17}^sH_{30}^sV_{11}^s \\
& + H_2V_9H_{15}^sH_{19}^sH_{22}^sV_{41}^s + H_2V_{10}H_3^sH_{15}^sH_{30}^sV_{11}^s + H_2V_{19}H_{11}H_{26}H_{19}^sV_{32}^s \\
& + H_2V_{30}\Phi_1\Phi_{23}H_{29}^sV_{49}^s + H_2V_{30}H_{29}^sV_{31}^sV_{32}^sV_{49}^s + H_2V_{39}H_{11}V_{37}H_3^sH_8^s \\
& + H_2V_{39}\Phi_3\Phi_3H_{21}^sV_{41}^s + H_2V_{39}\Phi_{12}\overline{\Phi}_{12}H_{21}^sV_{41}^s + H_2V_{39}\Phi_4'\overline{\Phi}_4'H_{21}^sV_{41}^s \\
& + H_2V_{39}H_3^sH_8^sH_{15}^sV_{49}^s + H_2V_{39}H_5^sH_{21}^sH_{30}^sV_{11}^s + H_2V_{39}H_{15}^sH_{18}^sH_{19}^sV_{41}^s \\
& + H_2V_{40}H_{11}V_{35}H_3^sH_8^s + H_2V_{40}V_5V_7H_{21}^sV_{41}^s + H_2V_{40}H_9^sH_{31}^sH_{39}^sV_{31}^s \\
& + H_2V_{40}H_{21}^sV_1^sV_2^sV_{41}^s + H_{25}H_{25}H_{26}H_{26}\Phi_{23}\overline{\Phi}_{56} + H_{25}H_{28}H_{23}H_{26}\Phi_{23}\overline{\Phi}_{56} \\
& + H_{25}H_{28}\Phi_1\overline{\Phi}_{56}H_3^sH_{10}^s + H_{25}H_{28}\Phi_{23}\overline{\Phi}_{56}H_6^sH_7^s + H_{25}V_9H_{13}H_{26}H_{15}^sV_{41}^s
\end{aligned}$$

$W_6(\text{hidden})$  continued:

$$\begin{aligned}
& +H_{25}V_9H_{26}V_{37}V_{41}^sV_{50}^s & +H_{25}V_{39}\Phi_3\overline{\Phi}_4'H_{30}^sV_{31}^s & +H_{25}V_{39}H_7^sH_8^sH_{30}^sV_{31}^s \\
& +H_{25}V_{39}H_8^sH_{15}^sH_{30}^sV_{47}^s & +H_{25}V_{39}H_9^sH_{10}^sH_{30}^sV_{31}^s & +H_{28}H_{28}H_{23}H_{23}\Phi_{23}\overline{\Phi}_{56} \\
& +H_{28}V_9H_{13}H_{23}H_{15}^sV_{41}^s & +H_{28}V_9H_{23}V_{37}V_{41}^sV_{50}^s & +H_{28}V_9V_{15}V_{37}H_3^sV_{48}^s \\
& +H_{28}V_{20}\Phi_1\overline{\Phi}_{56}H_7^sV_{44}^s & +H_{28}V_{20}H_{15}^sV_{32}^sV_{43}^sV_{48}^s & +H_{28}V_{30}H_{11}H_{13}H_{15}^sV_{32}^s \\
& +H_{28}V_{30}H_{11}V_{37}V_{32}^sV_{50}^s & +H_{28}V_{30}\Phi_2\overline{\Phi}_4H_{15}^sV_{32}^s & +H_{28}V_{30}H_3^sH_4^sH_{15}^sV_{32}^s \\
& +H_{28}V_{30}H_5^sH_6^sH_{15}^sV_{32}^s & +H_{28}V_{30}H_7^sV_{31}^sV_{32}^sV_{48}^s & +H_{28}V_{30}H_{15}^sV_{32}^sV_{41}^sV_{42}^s \\
& +H_{28}V_{30}H_{15}^sV_{32}^sV_{43}^sV_{44}^s & +H_{28}V_{30}H_{15}^sV_{32}^sV_{47}^sV_{48}^s & +H_{28}V_{30}H_{15}^sV_{32}^sV_{49}^sV_{50}^s \\
& +H_{28}V_{40}\Phi_2\overline{\Phi}_2H_{37}^sV_{32}^s & +H_{28}V_{40}\Phi_{13}\overline{\Phi}_{13}H_{37}^sV_{32}^s & +H_{28}V_{40}\Phi_4\overline{\Phi}_4H_{37}^sV_{32}^s \\
& +H_{28}V_{40}H_9^sV_{21}^sV_{32}^sV_{50}^s & +V_9V_{10}V_{19}V_{40}H_{21}^sH_{30}^s & +V_9V_{10}H_{23}V_{37}H_{30}^sV_{31}^s \\
& +V_9V_{10}H_5^sH_{10}^sH_{30}^sH_{38}^s & +V_9V_{19}H_{26}V_{37}H_3^sV_{48}^s & +V_9V_{19}\Phi_3\overline{\Phi}_3H_{17}^sH_{30}^s \\
& +V_9V_{19}\Phi_{12}\overline{\Phi}_{12}H_{17}^sH_{30}^s & +V_9V_{19}\Phi_4'\overline{\Phi}_4'H_{17}^sH_{30}^s & +V_9V_{19}H_{15}^sH_{19}^sH_{22}^sH_{30}^s \\
& +V_9V_{20}H_{26}V_{37}H_5^sV_{48}^s & +V_9V_{29}H_5^sH_{10}^sH_{19}^sH_{30}^s & +V_9V_{30}H_{26}V_{37}H_5^sV_{44}^s \\
& +V_9V_{40}H_4^sH_{21}^sV_2^sV_{41}^s & +V_9V_{40}H_8^sH_{29}^sV_{12}^sV_{49}^s & +V_9V_{40}H_9^sH_{32}^sV_{11}^sV_{48}^s \\
& +V_{10}V_{20}\Phi_3\overline{\Phi}_3H_{15}^sH_{30}^s & +V_{10}V_{20}\Phi_{12}\overline{\Phi}_{12}H_{15}^sH_{30}^s & +V_{10}V_{20}\Phi_4'\overline{\Phi}_4'H_{15}^sH_{30}^s \\
& +V_{10}V_{30}H_5^sH_{10}^sH_{21}^sH_{30}^s & +V_{19}V_{39}\Phi_3\overline{\Phi}_3H_{21}^sH_{30}^s & +V_{19}V_{39}\Phi_{12}\overline{\Phi}_{12}H_{21}^sH_{30}^s \\
& +V_{19}V_{39}\Phi_4'\overline{\Phi}_4'H_{21}^sH_{30}^s & +V_{19}V_{39}H_{15}^sH_{18}^sH_{19}^sH_{30}^s & +V_{19}V_{40}V_5V_7H_{21}^sH_{30}^s \\
& +V_{19}V_{40}H_7^sH_8^sH_{19}^sH_{31}^s & +V_{19}V_{40}H_9^sH_{10}^sH_{19}^sH_{31}^s & +V_{19}V_{40}H_{21}^sH_{30}^sV_1^sV_2^s \\
& +V_{20}V_{39}V_7V_{37}H_{15}^sH_{30}^s & +V_{20}V_{40}V_7V_{35}H_{15}^sH_{30}^s & +V_{29}V_{30}H_{29}^sH_{30}^sV_{31}^sV_{32}^s \\
& +V_{29}V_{39}H_3^sH_8^sH_{15}^sH_{30}^s & +V_{30}V_{40}H_5^sH_8^sH_{18}^sH_{29}^s & +H_{11}H_{13}H_{26}V_{27}H_{15}^sV_{32}^s \\
& +H_{11}H_{13}H_{26}V_{37}H_{37}^sV_{32}^s & +H_{11}H_{13}V_5V_{17}H_{15}^sH_{30}^s & +H_{11}H_{13}V_{15}V_{35}H_{21}^sH_{30}^s \\
& +H_{11}H_{13}H_{15}^sH_{30}^sV_2^sV_{11}^s & +H_{11}H_{26}V_{27}V_{37}V_{32}^sV_{50}^s & +H_{11}V_5V_{17}V_{37}H_{30}^sV_{50}^s \\
& +H_{11}V_5H_8^sH_{17}^sH_{30}^sV_{12}^s & +H_{11}V_{15}H_8^sH_{17}^sH_{30}^sV_2^s & +H_{11}V_{17}H_{15}^sH_{20}^sH_{30}^sV_{50}^s \\
& +H_{11}V_{35}H_{10}^sH_{19}^sH_{30}^sV_{12}^s & +H_{11}V_{37}H_8^sV_{32}^sV_{42}^sV_{43}^s & +H_{11}V_{37}H_{30}^sV_2^sV_{11}^sV_{50}^s \\
& +H_{13}H_{26}H_9^sH_{15}^sH_{16}^sH_{39}^s & +H_{13}H_{26}H_{15}^sH_{37}^sV_{32}^sV_{49}^s & +H_{13}V_5H_5^sH_{21}^sH_{30}^sV_{21}^s \\
& +H_{13}V_5H_{15}^sH_{31}^sH_{38}^sV_{41}^s & +H_{13}V_7H_7^sH_{15}^sH_{30}^sV_{11}^s & +H_{13}V_{17}H_7^sH_{15}^sH_{30}^sV_1^s \\
& +H_{13}V_{35}H_3^sH_{15}^sH_{30}^sV_{21}^s & +H_{23}H_{26}\Phi_1\overline{\Phi}_1H_5^sH_8^s & +H_{23}H_{26}\Phi_1\overline{\Phi}_{23}H_4^sH_9^s \\
& +H_{23}H_{26}\Phi_{23}\overline{\Phi}_{23}H_5^sH_8^s & +H_{23}H_{26}\Phi_{56}\overline{\Phi}_{56}H_5^sH_8^s & +H_{23}H_{26}\Phi_{56}'\overline{\Phi}_{56}'H_5^sH_8^s \\
& +H_{23}H_{26}H_4^sH_9^sV_{31}^sV_{32}^s & +H_{23}H_{26}H_8^sH_{29}^sV_{12}^sV_{41}^s & +H_{23}V_5V_7V_{37}H_{30}^sV_{31}^s \\
& +H_{23}V_7H_{15}^sH_{20}^sH_{30}^sV_{31}^s & +H_{23}V_{35}\Phi_3\overline{\Phi}_3H_{30}^sV_{31}^s & +H_{23}V_{35}\Phi_{12}\overline{\Phi}_{12}H_{30}^sV_{31}^s \\
& +H_{23}V_{35}\Phi_4'\overline{\Phi}_4'H_{30}^sV_{31}^s & +H_{23}V_{37}H_{30}^sV_1^sV_2^sV_{31}^s & +H_{26}V_5H_{21}^sV_{12}^sV_{41}^sV_{48}^s \\
& +H_{26}V_7H_{15}^sH_{30}^sH_{39}^sV_{12}^s & +H_{26}V_{15}\Phi_1\overline{\Phi}_1H_9^sV_{44}^s & +H_{26}V_{15}\Phi_{23}\overline{\Phi}_{23}H_9^sV_{44}^s \\
& +H_{26}V_{15}\Phi_{56}\overline{\Phi}_{56}H_9^sV_{44}^s & +H_{26}V_{15}\Phi_{56}'\overline{\Phi}_{56}'H_9^sV_{44}^s & +H_{26}V_{15}H_{21}^sV_2^sV_{41}^sV_{48}^s \\
& +H_{26}V_{17}H_{15}^sV_{32}^sV_{43}^sV_{48}^s & +H_{26}V_{25}H_9^sV_{31}^sV_{32}^sV_{48}^s & +H_{26}V_{27}\Phi_2\overline{\Phi}_4H_{15}^sV_{32}^s \\
& +H_{26}V_{27}H_3^sH_4^sH_{15}^sV_{32}^s & +H_{26}V_{27}H_5^sH_6^sH_{15}^sV_{32}^s & +H_{26}V_{27}H_7^sV_{31}^sV_{32}^sV_{48}^s \\
& +H_{26}V_{27}H_{15}^sV_{32}^sV_{41}^sV_{42}^s & +H_{26}V_{27}H_{15}^sV_{32}^sV_{43}^sV_{44}^s & +H_{26}V_{27}H_{15}^sV_{32}^sV_{47}^sV_{48}^s \\
& +H_{26}V_{27}H_{15}^sV_{32}^sV_{49}^sV_{50}^s & +H_{26}V_{37}\Phi_2\overline{\Phi}_4H_{37}^sV_{32}^s & +H_{26}V_{37}H_3^sH_4^sH_{37}^sV_{32}^s \\
& +H_{26}V_{37}H_5^sH_6^sH_{37}^sV_{32}^s & +H_{26}V_{37}H_9^sH_{16}^sH_{39}^sV_{50}^s & +H_{26}V_{37}H_{37}^sV_{32}^sV_{41}^sV_{42}^s \\
& +H_{26}V_{37}H_{37}^sV_{32}^sV_{43}^sV_{44}^s & +H_{26}V_{37}H_{37}^sV_{32}^sV_{47}^sV_{48}^s & +H_{26}V_{37}H_{37}^sV_{32}^sV_{49}^sV_{50}^s \\
& +V_5V_7H_5^sH_{10}^sH_{30}^sH_{38}^s & +V_5V_{17}H_3^sH_4^sH_{15}^sH_{30}^s & +V_5V_{17}H_5^sH_6^sH_{15}^sH_{30}^s \\
& +V_5V_{17}H_7^sH_{30}^sV_{31}^sV_{48}^s & +V_5V_{17}H_{15}^sH_{30}^sV_{41}^sV_{42}^s & +V_5V_{17}H_{15}^sH_{30}^sV_{43}^sV_{44}^s
\end{aligned}$$

$W_6(\text{hidden})$  continued:

$$\begin{aligned}
& +V_5 V_{17} H_{15}^s H_{30}^s V_{47}^s V_{48}^s & +V_5 V_{17} H_{15}^s H_{30}^s V_{49}^s V_{50}^s & +V_5 V_{25} H_9^s H_{30}^s V_{31}^s V_{44}^s \\
& +V_5 V_{27} H_7^s H_{30}^s V_{31}^s V_{44}^s & +V_5 V_{27} H_{15}^s H_{30}^s V_{44}^s V_{47}^s & +V_5 V_{37} H_{30}^s H_{37}^s V_{44}^s V_{47}^s \\
& +V_5 V_{37} H_{31}^s H_{38}^s V_{41}^s V_{50}^s & +V_7 V_{15} \Phi_3 \overline{\Phi}_4' H_{17}^s H_{30}^s & +V_7 V_{15} H_7^s H_8^s H_{17}^s H_{30}^s \\
& +V_7 V_{15} H_9^s H_{10}^s H_{17}^s H_{30}^s & +V_7 V_{37} H_6^s H_{15}^s V_{32}^s V_{43}^s & +V_7 V_{37} H_7^s H_{30}^s V_{11}^s V_{50}^s \\
& +V_{15} V_{35} \Phi_3 \overline{\Phi}_4' H_{21}^s H_{30}^s & +V_{15} V_{35} H_3^s H_4^s H_{21}^s H_{30}^s & +V_{15} V_{35} H_5^s H_6^s H_{21}^s H_{30}^s \\
& +V_{15} V_{35} H_{21}^s H_{30}^s V_{41}^s V_{42}^s & +V_{15} V_{35} H_{21}^s H_{30}^s V_{43}^s V_{44}^s & +V_{15} V_{35} H_{21}^s H_{30}^s V_{47}^s V_{48}^s \\
& +V_{15} V_{35} H_{21}^s H_{30}^s V_{49}^s V_{50}^s & +V_{15} V_{37} \Phi_3 \Phi_3 H_{19}^s H_{31}^s & +V_{15} V_{37} \Phi_{12} \overline{\Phi}_{12} H_{19}^s H_{31}^s \\
& +V_{15} V_{37} \Phi_4' \overline{\Phi}_4' H_{19}^s H_{31}^s & +V_{17} V_{37} H_7^s H_{30}^s V_1^s V_{50}^s & +V_{25} V_{27} H_{29}^s H_{30}^s V_{31}^s V_{32}^s \\
& +V_{25} V_{35} H_{21}^s H_{30}^s V_{41}^s V_{50}^s & +V_{35} V_{37} H_3^s H_{30}^s V_{21}^s V_{50}^s & 
\end{aligned} \tag{B.15}$$



## C Maximally Orthogonal $D$ -flat FNY Basis Set

$D$ - flat dir.	$\frac{Q^{(A)}}{112}$	$\Phi_{12}$ $H_{29}^s$ $V_2^s$	$\Phi_{13}$ $H_{17}^s$ $V_{11}^s$	$\Phi_{23}$ $V_{31}^s$ $V_{22}^s$	$\overline{\Phi}_{56}$ $V_{32}^s$ $V_{12}^s$	$\Phi'_{56}$ $H_{32}^s$ $H_{31}^s$	$\Phi_4^{(')}$ $V_{21}^s$ $H_{18}^s$	$H_{30}^s$ $H_{22}^s$ $N_2^c$	$H_{19}^s$ $H_{36}^s$ $N_1^c$	$H_{21}^s$ $H_{37}^s$ $H_{39}^s$	$H_{38}^s$ $H_{20}^s$ $H_{16}^s$	$H_{15}^s$ $V_1^s$ $N_3^c$
$K_1$	0	1-1										
$K_2$	0		1-1									
$K_3$	0			1-1								
$K_4$	0				1-1							
$K_5$	0					1-1						
$K_6$	0						1-1					
$K_7$	0						1-1'					
$K_8$	0						1'-1					
$L_1$	-2	0	8	7	2	3	2	0	0	0	0	0
		0	0	0	0	0	2	0	0	0	0	0
		0	0	0	0	0	0	0	6	-2	2	-4
$L_2$	-2	0	8	6	3	2	3	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	2	0	6	0	2	-4
$L_3$	-2	0	8	7	2	3	2	0	0	0	0	0
		0	0	0	0	0	0	0	0	2	0	0
		0	0	0	0	0	0	0	6	0	2	-6
$L_4$	-2	0	8	7	2	3	2	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	2
		0	0	0	0	0	0	0	8	-2	2	-6
$L_5$	-2	0	6	5	3	2	2	0	0	2	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	4	0	2	-4
$L_6$	-2	0	6	5	2	1	0	0	0	0	2	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	2	0	2	-2
$L_7$	-2	0	8	5	2	3	0	0	0	0	0	0
		0	0	2	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	6	0	4	-4
$L_8$	-2	0	6	5	2	1	0	0	2	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	4	0	2	-4
$L_9$	-1	0	4	3	1	1	1	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	1	0	0	0	0	0	3	-1	1	-2

$D-$ flat dir.	$\frac{Q^{(A)}}{112}$	$\Phi_{12}$ $H_{29}^s$ $V_2^s$	$\Phi_{13}$ $H_{17}^s$ $V_{11}^s$	$\Phi_{23}$ $V_{31}^s$ $V_{22}^s$	$\overline{\Phi}_{56}$ $V_{32}^s$ $V_{12}^s$	$\Phi'_{56}$ $H_{32}^s$ $H_{31}^s$	$\Phi_4^{(\prime)}$ $V_{21}^s$ $H_{18}^s$	$H_{30}^s$ $H_{22}^s$ $N_2^c$	$H_{19}^s$ $H_{36}^s$ $N_1^c$	$H_{21}^s$ $H_{37}^s$ $H_{39}^s$	$H_{38}^s$ $H_{20}^s$ $H_{16}^s$	$H_{15}^s$ $V_1^s$ $N_3^c$
$L_{10}$	0	0	0	0	1	1	-2	0	0	0	0	0
		0	2	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	2	0
$L_{11}$	0	0	0	1	1	2	0	0	0	0	0	0
		2	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	2	0	2	-2
$L_{12}$	0	1	-1	-1	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
$L_{13}$	0	0	0	1	1	0	2	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	2	0	0	2	0	-2	-2
$L_{14}$	0	0	0	1	0	1	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	2	0	0	0	-2
$L_{15}$	0	0	0	0	0	1	0	0	0	0	0	1
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	1	0
$L_{16}$	0	0	-2	-1	-1	-2	0	2	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	-2	0	-2	2
$L_{17}$	1	0	-3	-3	-1	-1	-1	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	0	1	0	0	0	-3	1	-1	2
$L_{18}$	2	0	-6	-5	-2	-3	-2	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		0	0	2	0	0	0	0	-6	2	-2	4
$L_{19}$	2	0	-8	-7	-2	-3	0	0	0	0	0	0
		0	0	0	2	0	0	0	0	0	0	0
		0	0	0	0	0	0	0	-6	0	-4	4
$L_{20}$	2	0	-6	-5	-2	-3	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	2	0
		0	0	0	0	0	0	0	-4	0	-2	4
$L_{21}$	2	0	-6	-5	-1	-2	0	0	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0
		2	0	0	0	0	0	0	-4	2	-2	2
$L_{22}$	2	0	-6	-5	-2	-3	-2	0	0	0	0	0
		0	0	0	0	0	0	0	2	0	0	0
		0	0	0	0	0	0	0	-6	0	-2	6

$D$ - flat dir.	$\frac{Q^{(A)}}{112}$	$\Phi_{12}$ $H_{29}^s$ $V_2^s$	$\Phi_{13}$ $H_{17}^s$ $V_{11}^s$	$\Phi_{23}$ $V_{31}^s$ $V_{22}^s$	$\overline{\Phi}_{56}$ $V_{32}^s$ $V_{12}^s$	$\Phi'_{56}$ $H_{32}^s$ $H_{31}^s$	$\Phi_4^{(\prime)}$ $V_{21}^s$ $H_{18}^s$	$H_{30}^s$ $H_{22}^s$ $N_2^c$	$H_{19}^s$ $H_{36}^s$ $N_1^c$	$H_{21}^s$ $H_{37}^s$ $H_{39}^s$	$H_{38}^s$ $H_{20}^s$ $H_{16}^s$	$H_{15}^s$ $V_1^s$ $N_3^c$
$L_{23}$	2	0	-6	-6	-1	-1	-2	0	0	0	0	0
		0	0	0	0	2	0	0	0	0	0	0
		0	0	0	0	0	0	0	-4	2	0	4
$L_{24}$	2	0	-6	-5	-1	-2	-2	0	0	0	0	0
		0	0	0	0	0	0	2	0	0	0	0
		0	0	0	0	0	0	0	-4	0	-2	4

Table C.I: Maximally orthogonal basis set of singlet VEVs directions that are  $D$ -flat for all non-anomalous  $U(1)$  local symmetries and do not break  $U(1)_Y$ . The first entry specifies the designation for a flat direction, while the second entry gives the anomalous charge associated with it. The remaining entries give the ratios of the norms of the component VEVs in each direction. A “1 – 1” entry for a  $K_j$  flat direction component from field  $\Phi_m$  implies that  $|\langle \Phi_m \rangle|^2 = |\langle \overline{\Phi}_m \rangle|^2$ .

$D$ - flat dir.	$\frac{Q^{(A)}}{112}$	$\Phi_{12}$	$\Phi_{13}$	$\Phi_{23}$	$\overline{\Phi}_{56}$	$\Phi'_{56}$	$\Phi_4^{(\prime)}$	$V_{31}^s$	$V_{32}^s$	$V_{21}^s$	$V_1^s$	$V_2^s$	$V_{11}^s$	$V_{22}^s$	$V_{12}^s$
$L'_1$	0	1	-1	-1	0	0	0	0	0	0	0	0	0	0	0
$L'_2$	0	0	0	-1	0	0	0	1	1	0	0	0	0	0	0
$L'_3$	0	0	1	0	0	0	0	0	0	0	0	0	1	0	1
$L'_4$	0	0	1	1	0	0	0	0	0	1	0	0	0	1	0
$L'_5$	0	0	2	1	0	1	0	0	0	2	0	0	0	0	2

Table C.II: Maximally orthogonal basis set of non-trivial singlet VEVs directions that are  $D$ -flat for all non-anomalous  $U(1)$  local symmetries and that break neither  $U(1)_Y$  nor  $U(1)_{Z'}$ . None of these directions carry anomalous charge  $Q^{(A)}$

## D $F$ -flat Directions

$c\#$	# VEVs	$\mathcal{O}(\mathcal{S})$ term	$\{VEV_1\}$	$\bar{\Phi}_{56}$	$\bar{\Phi}'_{56}$	$H_{19}^s$	$V_{31}^s$	$H_{20}^s$	$N_1^c$	$N_3^c$	$H_{21}^s$	$H_{17}^s$	$V_{12}^s$	$H_{18}^s$	$H_{39}^s$
1	8+3	$\infty$	*	*											
2	9+3	$\infty$	*	*	$\bar{*}$										
3	9+3	$\infty$	*	*		*									
4	9+3	12-1,2	*	*				*							
5	10+3	7-1	*	*			*		*						
6	11+3	7-1	*	*	$\bar{*}$		*		*						
7	11+3	7-1	*	*		*	*		*						
8	11+3	7-1	*	*			*	*	*						
9	10+3	6-1	*	*			*			*					
10	11+3	6-1	*	*			*		*	*					
11	11+3	6-1	*	*			*	*		*					
12	12+3	6-1	*	*			*	*	*	*					
13	11+3	6-1	*	*	$\bar{*}$		*			*					
14	9+3	6-1	*				*			*					
15	11+3	6-1	*				*	*	*	*					
16	12+3	6-1,2	*				*		*	*			*		*
17	10+3	6-1	*				*		*	*					
18	10+3	6-1	*				*			*		*			
19	10+3	6-1	*				*			*				*	
20	11+3	6-1	*		$\bar{*}$		*		*	*					
21	12+3	6-1	*		$\bar{*}$		*		*	*				*	
22	10+3	6-1	*		$\bar{*}$		*			*					
23	11+3	6-1	*		$\bar{*}$		*			*	*				
24	12+3	6-1,2	*		$\bar{*}$		*			*			*		*
25	10+4	6-1	*		$\bar{*}$		*			*		*			
26	11+4	6-1	*		$\bar{*}$		*		*	*		*			
27	11+3	6-1	*	$\bar{*}$			*	*		*					
28	12+3	6-1,2	*	$\bar{*}$			*			*			*		*
29	12+3	6-1	*	$\bar{*}$	$\bar{*}$		*		*	*					
30	11+3	6-1	*	$\bar{*}$	$\bar{*}$		*			*					

Table D.I: Classes of  $D$ -flat directions producing MSSM massless field content that are  $F$ -flat to at least  $6^{th}$  order in the superpotential. These classes are defined by their respective set of non-Abelian singlet states that develop VEVs and are identified by their class identification number appearing in column one. Column two indicates the number of singlets that acquire VEVs in each direction, with the first entry being the number of states that take on VEVs as a result of FI cancellation and the

second entry being the number of  $\Phi_4$ -related states for which we additionally require VEVs. Column three gives the order in the FNY superpotential at which flatness is broken and the designation of the superpotential term in Table D.II responsible for the breaking. For example, a flat direction belonging to class 5 is broken by the seventh order term designated 7-1.

The remaining columns indicate which states receive VEVs. These are indicated by the \*'s in the respective columns. An \* implies the given state takes on a VEV, while a  $\bar{*}$  implies the vector partner instead.  $\{VEV_1\}$  denotes the set of states  $\{\Phi_{12}, \Phi_{23}, (\Phi_4, \Phi'_4, \bar{\Phi}_4, \bar{\Phi}'_4), H_{30}^s, H_{38}^s, H_{15}^s, H_{31}^s\}$ . An \* in the  $\{VEV_1\}$  column implies that all of the states in  $\{VEV_1\}$  receive VEVs. Furthermore,  $*'$  in the  $\{VEV_1\}$  column indicates that, while present, none of the  $\Phi_4$  VEVs are required for FI-term cancellation.

designation	FNY Superpotential Term
6-1	$H_{15}^s H_{30}^s H_{31}^s H_{36}^s V_{31}^s N_3^c$
6-2	$H_{15}^s H_{30}^s H_{31}^s H_{39}^s V_{31}^s V_{22}^s$
7-1	$\Phi_4' H_{15}^s H_{20}^s H_{30}^s H_{31}^s V_{31}^s N_1^c$
12-1	$\Phi_{23} \bar{\Phi}_{56} \Phi_4'^2 H_{15}^{s\,2} H_{20}^{s\,2} H_{31}^{s\,2} H_{38}^{s\,2}$
12-2	$\Phi_{23} \bar{\Phi}_{56} \Phi_4'^2 H_{15}^{s\,2} H_{20}^{s\,2} H_{31}^{s\,2} H_{38}^{s\,2}$

Table D.II: FNY superpotential terms that break  $F$ -flatness of  $D$ -flat directions at sixth order or higher. The first column gives a designation for each term.

$c\#$	$\frac{Q^{(A)}}{112}$	$\{\Phi_{12}, \Phi_{23}, (\Phi_4), H_{30}^s H_{38}^s, H_{15}^s, H_{31}^s, \}$	$\overline{\Phi}_{56}$ $N_3^c$	$\Phi'_{56}$ $H_{21}^s$	$H_{19}^s$ $H_{17}^s$	$V_{31}^s$ $V_{12}^s$	$H_{20}^s$ $H_{18}^s$	$N_1^c$ $H_{39}^s$
1	-2	3, 1, 1, 3, 2, 2, 1	1	0	0	0	0	0
			0	0	0	0	0	0
2	-6	10, 2, 2, 8, 6, 4, 2	3	-1	0	0	0	0
			0	0	0	0	0	0
3	-4	4, 3, 2, 8, 2, 6, 2	1	0	2	0	0	0
			0	0	0	0	0	0
4	-2	3, 2, 3, 3, 4, 4, 3	2	0	0	0	2	0
			0	0	0	0	0	0
5	-4	6, 1, 2, 8, 2, 4, 2	1	0	0	2	0	2
			0	0	0	0	0	0
6	-6	10, 1, 2, 10, 4, 4, 2	2	-1	0	2	0	2
			0	0	0	0	0	0
7	-8	10, 3, 4, 18, 2, 10, 4	1	0	2	4	0	4
			0	0	0	0	0	0
8	-4	6, 1, 4, 10, 2, 6, 4	1	0	0	4	2	4
			0	0	0	0	0	0
9	-4	5, 2, 2, 8, 3, 5, 2	1	0	0	1	0	0
			1	0	0	0	0	0
10	-8	10, 3, 4, 18, 4, 10, 4	1	0	0	4	0	2
			2	0	0	0	0	0
11	-4	4, 3, 4, 10, 4, 8, 4	1	0	0	2	2	0
			2	0	0	0	0	0
12	-4	4, 3, 6, 12, 4, 10, 6	1	0	0	4	4	2
			2	0	0	0	0	0
13	-6	8, 2, 2, 12, 4, 6, 2	1	-1	0	2	0	0
			2	0	0	0	0	0
14	-2	2, 1, 1, 5, 1, 3, 1	0	0	0	1	0	0
			1	0	0	0	0	0
25	-4	4, 2, 4, 12, 2, 8, 4	0	0	0	4	2	2
			2	0	0	0	0	0
16	-7	9, 1, 3, 18, 2, 9, 4	0	0	0	6	0	3
			2	0	0	1	0	1
17	-4	5, 1, 2, 10, 1, 5, 2	0	0	0	3	0	2
			1	0	0	0	0	0
18	-3	2, 2, 1, 9, 1, 5, 2	0	0	0	2	0	0
			2	0	1	0	0	0
19	-6	6, 3, 5, 16, 2, 10, 2	0	0	0	2	0	0
			4	0	0	0	2	0
20	-6	8, 1, 2, 14, 2, 6, 2	0	-1	0	4	0	2
			2	0	0	0	0	0

$c\#$	$\frac{Q^{(A)}}{112}$	$\{\Phi_{12}, \Phi_{23}, (\Phi_4), H_{30}^s H_{38}^s, H_{15}^s, H_{31}^s\}$	$\bar{\Phi}_{56}$ $N_3^c$	$\Phi'_{56}$ $H_{21}^s$	$H_{19}^s$ $H_{17}^s$	$V_{31}^s$ $V_{12}^s$	$H_{20}^s$ $H_{18}^s$	$N_1^c$ $H_{39}^s$
21	-8	10, 2, 5, 20, 2, 10, 2	0 4	-1 0	0 0	4 0	0 2	2 0
22	-4	5, 1, 1, 9, 2, 4, 1	0 2	-1 0	0 0	2 0	0 0	0 0
23	-8	8, 3, 4, 20, 2, 10, 2	0 4	-1 2	0 0	4 0	0 0	0 0
24	-8	10, 1, 2, 20, 4, 10, 4	0 4	-1 0	0 0	6 2	0 0	0 2
25	-3	3, 1, 0, 8, 1, 3, 1	0 2	-1 0	0 1	2 0	0 0	0 0
26	-8	10, 1, 0, 20, 2, 6, 2	0 4	-3 0	0 2	6 0	0 0	2 0
27	-4	2, 3, 4, 14, 2, 10, 4	-1 4	0 0	0 0	4 0	2 0	0 0
28	-6	6, 1, 2, 18, 2, 10, 4	-1 4	0 0	0 0	6 2	0 0	0 2
29	-8	10, 1, 2, 20, 2, 8, 2	-1 4	-2 0	0 0	6 0	0 0	2 0
30	-6	6, 2, 2, 16, 2, 8, 2	-1 4	-1 0	0 0	4 0	0 0	0 0

Table D.III: Examples of flat directions from the various classes presented in Table D.I.a. Column one indicates the class of directions to which the example belongs. Column two gives the anomalous charge  $Q^{(A)}/112$  of the example flat direction. The remaining column entries specify the ratios of the norms of the VEVs in the flat direction.  $\{\Phi_{12}, \Phi_{23}, (\Phi_4), H_{30}^s H_{38}^s, H_{15}^s, H_{31}^s\} \equiv \{VEV_1\}$  is the set of the norms that are non-zero for all flat directions. The third component VEV, involving all of the  $\Phi_4$ -related states, is the net value of  $|\langle\Phi_4\rangle|^2 + |\langle\Phi'_4\rangle|^2 - |\langle\bar{\Phi}_4\rangle|^2 - |\langle\bar{\Phi}'_4\rangle|^2$ . Thus, a “0” in the  $\Phi_4$  column for classes 14 and 27 implies that  $|\langle\Phi_4\rangle|^2 + |\langle\Phi'_4\rangle|^2 - |\langle\bar{\Phi}_4\rangle|^2 - |\langle\bar{\Phi}'_4\rangle|^2 = 0$ , while each of the four states still takes on a VEV.



c#	# MOBD	MOBD expression
1	5+2	$2L_6 + 6L_{12} + L_{13} + 4L_{15} + 3L_{16} + K_7 + K_8$
2	5+2	$3L_6 + 10L_{12} + L_{13} + 4L_{15} + 4L_{16} + K_7 + K_8$
3	6+2	$L_6 + 4L_{12} + L_{13} + 6L_{15} + 4L_{16} + L_8 + K_7 + K_8$
4	6+2	$3L_6 + 6L_{12} + 2L_{13} + 6L_{15} + 3L_{16} + L_{20} + K_7 + K_8$
5	6+2	$L_6 + 6L_{12} + L_{13} + 4L_{15} + 4L_{16} + L_7 + K_7 + K_8$
6	6+2	$2L_6 + 10L_{12} + L_{13} + 4L_{15} + 5L_{16} + L_7 + K_7 + K_8$
7	7+2	$L_6 + 10L_{12} + 2L_{13} + 10L_{15} + 9L_{16} + 2L_7 + L_8 + K_7 + K_8$
8	7+2	$L_6 + 6L_{12} + 2L_{13} + 6L_{15} + 5L_{16} + 2L_7 + L_{20} + K_7 + K_8$
9	6+2	$3L_6 + 10L_{12} + 2L_{13} + 10L_{15} + 8L_{16} + L_7 + K_7 + K_8$
10	6+2	$2L_6 + 10L_{12} + 2L_{13} + 10L_{15} + 9L_{16} + 2L_7 + K_7 + K_8$
11	7+2	$2L_6 + 4L_{12} + 2L_{13} + 8L_{15} + 5L_{16} + L_7 + L_{20} + K_7 + K_8$
12	7+2	$2L_6 + 4L_{12} + 3L_{13} + 10L_{15} + 6L_{16} + 2L_7 + 2L_{20} + K_7 + K_8$
13	6+2	$2L_6 + 8L_{12} + L_{13} + 6L_{15} + 6L_{16} + L_7 + K_7 + K_8$
14	6+2	$L_6 + 4L_{12} + L_{13} + 6L_{15} + 5L_{16} + L_7 + K_7 + K_8$
15	7+2	$L_6 + 4L_{12} + 2L_{13} + 8L_{15} + 6L_{16} + 2L_7 + L_{20} + K_7 + K_8$
16	7+2	$L_6 + 9L_{12} + 2L_{13} + 9L_{15} + 9L_{16} + 3L_7 + L_{17} + K_7 + K_8$
17	6+2	$L_6 + 10L_{12} + 2L_{13} + 10L_{15} + 10L_{16} + 3L_7 + K_7 + K_8$
18	7+2	$L_6 + 4L_{12} + 2L_{13} + 10L_{15} + 9L_{16} + 2L_7 + L_{10} + K_7 + K_8$
19	7+2	$L_6 + 6L_{12} + L_{13} + 10L_{15} + 8L_{16} + L_7 + L_2 + K_7 + K_8$
20	6+2	$L_6 + 8L_{12} + L_{13} + 6L_{15} + L_{16} * 7 + 2L_7 + K_7 + K_8$
21	7+2	$L_6 + 10L_{12} + L_{13} + 10L_{15} + 10L_{16} + 2L_7 + L_2 + K_7 + K_8$
22	6+2	$2L_6 + 10L_{12} + L_{13} + 8L_{15} + 9L_{16} + 2L_7 + K_7 + K_8$
23	7+2	$L_6 + 8L_{12} + L_{13} + 10L_{15} + 10L_{16} + 2L_7 + L_5 + K_7 + K_8$
24	7+2	$2L_6 + 10L_{12} + 2L_{13} + 10L_{15} + 10L_{16} + 3L_7 + 2L_{17} + K_7 + K_8$
25	7+2	$L_6 + 6L_{12} + L_{13} + 6L_{15} + 8L_{16} + 2L_7 + L_{10} + K_7 + K_8$
26	7+2	$L_6 + 10L_{12} + L_{13} + 6L_{15} + 10L_{16} + 3L_7 + L_{10} + K_7 + K_8$
27	7+2	$L_6 + 2L_{12} + 2L_{13} + 10L_{15} + 7L_{16} + 2L_7 + L_{20} + K_7 + K_8$
28	7+2	$L_6 + 6L_{12} + 2L_{13} + 10L_{15} + 9L_{16} + 3L_7 + 2L_{17} + K_7 + K_8$
29	6+2	$L_6 + 10L_{12} + L_{13} + 8L_{15} + 10L_{16} + 3L_7 + K_7 + K_8$
30	6+2	$L_6 + 6L_{12} + L_{13} + 8L_{15} + 8L_{16} + 2L_7 + K_7 + K_8$

Table D.IV: The flat direction in Table D.III expressed as linear combinations of the one-dimensional maximally orthogonal basis directions (MOBD) presented in Table D.I. Column one gives the class number of the flat direction. Column two specifies the number of unique MOBD's associated with each flat direction, the first entry being the number of  $L$ -class basis directions and the second entry the number of  $K$ -class. Column three gives the linear combination. Since the  $K$ -class terms contribution to each  $D$ -term is zero, their coefficients are not specified.

$c\#$	$\#$	$\frac{Q^{(A)}}{112}$	$\{\Phi_{12}, \Phi_{23}, (\Phi_4), H_{30}^s H_{38}^s, H_{15}^s, H_{31}^s, \}$	$\overline{\Phi}_{56}$ $N_3^c$	$\Phi'_{56}$ $H_{21}^s$	$H_{19}^s$ $H_{17}^s$	$V_{31}^s$ $V_{12}^s$	$H_{20}^s$ $H_{18}^s$	$N_1^c$ $H_{39}^s$
$X$	8	-2	3, 1, 1, 3, 2, 2, 1	1	0	0	0	0	0
$Y$	9	-2	2, 1, 1, 5, 1, 3, 1	0	0	0	1	0	0
$A$	7	0	0, 1, 2, 0, 2, 2, 2	1	0	0	0	2	0
$B$	7	-2	1, 2, 1, 5, 0, 4, 1	0	0	2	0	0	0
$C$	7	-2	3, 0, 1, 5, 0, 2, 1	0	0	0	2	0	2
$D$	7	-2	3, 0, 0, 4, 1, 1, 0	0	-1	0	1	0	0
$E$	9	-2	0, 3, 5, 9, 3, 9, 5	0	0	0	3	4	0
$F$	9	-4	0, 3, 4, 18, 0, 12, 4	-3	0	0	6	2	0
$G$	7	0	0, 0, 1, 1, 0, 1, 1	0	0	0	1	1	1
$H$	5	-2	4, 0, 0, 2, 2, 0, 0	1	-1	0	0	0	0
$I$	7	-1	0, 1, 0, 4, 0, 2, 1	0	0	0	1	0	0
$J$	7	-2	2, 1, 3, 6, 0, 4, 0	0	0	0	0	0	0
$M$	10	-4	5, 0, 1, 11, 2, 6, 3	0	0	0	4	0	0
$N$	7	-2	2, 0, 0, 6, 0, 2, 0	-1	-1	0	2	0	0
$P$	9	-2	1, 1, 1, 7, 0, 4, 1	-1	0	0	2	0	0
$Q$	9	-4	3, 2, 3, 11, 0, 6, 1	0	0	0	2	0	0
$R$	9	-3	4, 0, 0, 8, 0, 2, 1	0	-1	0	3	0	2
				1	0	1	0	0	0

Table D.V: Some one-dimensional physical directions that are  $D$ -flat for all non-anomalous  $U(1)_i$  and from which all  $D$ - and  $F$ -flat directions in Table D.IV may be formed.

c#	physical basis expression	# VEV fields	Dim <sub>FI</sub>	# U(1) broken
1	$X$	8	0	$7 + 1$
2	$2X + H$	9	1	$7 + 1$
3	$X + B$	9	1	$7 + 1$
4	$X + A$	9	1	$7 + 1$
5	$X + C$	10	1	$8 + 1$
6	$X + C + H$	11	2	$8 + 1$
7	$X + 2C + B$	11	2	$8 + 1$
8	$X + C + 2G$	11	2	$8 + 1$
9	$Y + C$	10	1	$8 + 1$
10	$Y$	9	0	$8 + 1$
11	$Y + D$	10	1	$8 + 1$
12	$X + Y$	10	1	$8 + 1$
13	$Y + I$	10	1	$8 + 1$
14	$D + I$	10	1	$8 + 1$
15	$2Y + J$	10	1	$8 + 1$
16	$2Y + N$	11	1	$9 + 1$
17	$3Y + E + F$	11	2	$8 + 1$
18	$X + Y + D$	11	2	$8 + 1$
19	$2X + Y + E$	11	2	$8 + 1$
20	$X + Y + C$	11	2	$8 + 1$
21	$Y + D + Q$	11	2	$8 + 1$
22	$Y + D + C$	11	2	$8 + 1$
23	$Y + G$	11	1	$9 + 1$
24	$Y + C + D + N$	12	3	$8 + 1$
25	$M + P$	12	1	$10 + 1$
26	$2X + Y + E + 4G$	12	3	$8 + 1$
27	$2D + I + R$	12	2	$9 + 1$
28	$Y + D + M$	12	2	$9 + 1$
29	$Y + C + D + J$	12	3	$8 + 1$
30	$2Y + 3C + 2M$	12	2	$9 + 1$

Table D.VI: Expressions for flat direction examples given in terms of physical dimension-one directions. Column two expresses the example flat direction for the class, indicated by the column one entry, in terms of the one-dimensional  $D$ -flat directions of Table D.V. Column three indicates the number of *independent* VEVs (which excludes the  $\Phi'_4$ ,  $\overline{\Phi}_4$ , and  $\overline{\Phi}'_4$  VEVs). Column four specifies the dimension of each flat direction (excluding the two degrees of freedom in the  $\overline{\Phi}_4$ , and  $\overline{\Phi}'_4$  VEVs) following cancellation of the FI term, which reduces a dimension by 1. This dimension is one less than the number of physical dimension-one  $D$ -flat-directions in column two since, prior to FI term cancellation, the degrees of freedom in a given class of flat directions are the coefficients  $w_{j,k}$  of the physical flat directions, in (3.14), associated

with a given class. The number of  $U(1)_i$  broken by a flat direction, including the anomalous  $U(1)_A$ , is specified in column five. This equals the difference between the number of independent VEVs in column three and the number of degrees of freedom in column four.

Since the gauge charges of  $\Phi'_4$ ,  $\overline{\Phi}_4$ , and  $\overline{\Phi}'_4$  are not independent from those of  $\Phi_4$ , the related terms  $K_7$  and  $K_8$ , that contain  $\Phi'_4$ ,  $\overline{\Phi}_4$ , and  $\overline{\Phi}'_4$  VEVs do not offer independent degrees of freedom that result in additional breaking of non-anomalous  $U(1)_i$ . Therefore, these terms are excluded in column two, as are their two degrees of freedom in column four.

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